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Nonlocal conductivity effects in low-pressure cylindrical inductive discharges

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Abstract. The study is on nonlocal conductivity and stochastic heating in low-pressure inductive discharges with cylindrical coils. The rf current density derived by solving the Boltzmann equation in cylindrical coordinates, with accounting for both the high-frequency heating field and the dc field in the discharge, is coupled with the wave equation resulting into the spatial distribution – under the conditions of anomalous skin – of the electromagnetic field components, of the rf current density and the power deposition in the free-fall regime of maintenance of hydrogen discharges.

1. Introduction

The inductive discharges [1] have always focused attention not only due to their use as plasma sources in the gas-discharge applications but also because they are rich with plenty of curious phenomena attractive for basic research. The power deposition in the discharge is such a phenomenon and its study combines gas-discharge physics with wave physics.

The inductive discharges are produced by transverse high-frequency waves under the conditions of wave field penetration over the skin depth. When the discharge is at low gas pressures, in the free-fall regime, the skin is anomalous [2,3] due to the large mean free path of the electrons. The electrons gain momentum in the rf heating field close to the discharge walls and transfer it – through their thermal motion – into the plasma interior. The local relation $\mathbf{j} = \sigma\mathbf{E}$ between current density $(\mathbf{j})$ and electric field $(\mathbf{E})$ is not valid anymore and the current density at a given position is determined by the electric field over the entire trajectory of the electron on the distance of its mean free path: $\mathbf{j}(t,\mathbf{r}) = \iint \sigma(t,t',\mathbf{r},\mathbf{r}')\mathbf{E}(t',\mathbf{r}')d\mathbf{r}'dt'$ where $\sigma(t,t',\mathbf{r},\mathbf{r}')$ is the conductivity kernel. Respectively, the electron heating [1,4,5] in the field is stochastic (collisionless) and the description is within the kinetic plasma theory.

The research on the rf power deposition in low-pressure inductive discharges [4-6] has revived the studies [7-9] on anomalous skin and nonlocal conductivity in the late 60's and the early 70's and involved them in the discharge modelling. However, the work up to now has covered only inductive discharges with planar coils, i.e. a planar configuration regarding to the wave penetration into the plasma. Anomalous skin in plasmas with a cylindrical configuration have been studied [9, 10], however, as a "pure" wave phenomenon only and, moreover, in approximations.

This study provides derivation of the nonlocal current density in cylindrical inductive discharges sustained at low gas pressure (i.e. under the conditions of anomalous skin) as well as numerical results on the spatial distribution of the field components of the wave sustaining the discharge, of the rf current density and of the wave power deposition into the discharge. The deep penetration of the wave field and of the current density into the plasma and the appearance of spatial regions of negative power deposition into the discharge are obtained as effects due to the importance of the thermal motion of the electrons.

2. Analytical derivation of the nonlocal current density in cylindrical inductive discharges

Inductive discharges (of radius R) with cylindrical coils are considered (figure 1). The azimithal rf current in the coil forms the configuration (an azimuthal electric field $\tilde{E}_\varphi$ and an axial magnetic field $\tilde{H}_z$) of the transverse high-frequency wave (of frequency ω) which sustains the discharge. Obtaining both the spatial distribution of the amplitude $E_\varphi(r)$ of the high-frequency field $(\tilde{E}_\varphi(r,t) = E_\varphi(r)\exp(-\mathrm{i}\omega t))$ from

$$\frac{\mathrm{d}^2 E_\varphi(r)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}E_\varphi}{\mathrm{d}r} + \left(\frac{\omega^2}{c^2} - \frac{1}{r^2}\right)E_\varphi(r) = -\mathrm{i}\omega\mu_0 j_\varphi(r) \qquad (1)$$

and the power deposition $Q = \mathbf{j}\cdot\tilde{\mathbf{E}}$ into the discharge requires to have the wave equation (1) coupled with an expression for the rf current density j_φ in the plasma; in (1), c is the light speed in vacuum and μ_0 is the vacuum susceptibility.

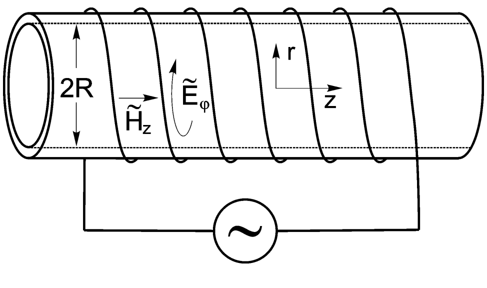

Figure 1. Configuration of the discharge.

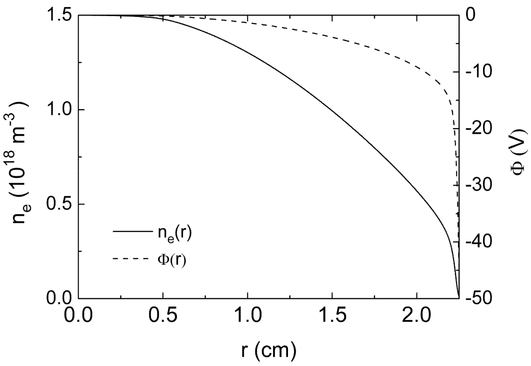

Figure 2. Radial profiles of the plasma density n_e and of the potential Φ of the radial dc field: a hydrogen discharge at $p = 7$ mTorr.

According to the maintenance of low pressure inductive discharges, (i) the wave is under the conditions of reflection from the plasma, i.e. it penetrates into the plasma over the skin depth, producing overdense plasmas ($n_e > n_{\mathrm{cr}}$, respectively, $\omega < \omega_{\mathrm{pl}}$ where n_{cr} is the critical density and ω_{pl} is the plasma frequency) and, (ii) there is a radial dc space charge field in the discharge which forms a potential well for the electrons. Therefore, in order to obtain j_φ we should know the radial profiles of the electron concentration $n_e(r)$ and of the potential $\Phi(r)$ of the dc field. In the presentation of the numerical results in the next section, the radial distribution of n_e and Φ is that (figure 2) obtained within the fluid-plasma model [11] of the free-fall regime of maintenance of hydrogen discharges.

For deriving the general expression for the rf current density which covers the case of an anomalous skin and nonlocal conductivity, of interest for the description of the low pressure inductive discharges, we should start from the Boltzmann equation written in cylindrical (r,φ,z) co-ordinates:

$$\frac{\partial f}{\partial t}+v_r\frac{\partial f}{\partial r}+\left(\frac{v_\varphi^2}{r}+\frac{e}{m_e}\frac{\partial\Phi(r)}{\partial r}\right)\frac{\partial f}{\partial v_r}-\left(\frac{v_r v_\varphi}{r}+\frac{e\tilde{E}_\varphi(r,t)}{m_e}\right)\frac{\partial f}{\partial v_\varphi}=\frac{\delta f}{\delta t}. \qquad (2)$$

Here $f(r,\mathbf{v},t)$ is the electron velocity distribution function, v_r and v_φ are, respectively, the radial and azimuthal velocities of the electrons and m_e is their mass; $\partial/\partial\varphi=0$ and $\partial/\partial z=0$ hold according to, respectively, azimuthal symmetry present and edge effects neglected. For obtaining the current density we need the solution of the Boltzmann equation for the oscillating part $\tilde{f}_1(r,\mathbf{v},t)=f_1(r,\mathbf{v})\exp(-i\omega t)$ of the distribution function. A Maxwellian distribution is assumed for the isotropic part $f_0(r,\mathbf{v})$ of the distribution function and the Bhatnagar-Gross-Krook approximation $\delta f/\delta t\approx-\nu_{e-n}(f-f_0)$ is employed for the collision integral; ν_{e-n} is the electron-neutral elastic collision frequency.

After specifying the sign of the radial component of the electron velocity by introducing $f_{1+}=f_1(v_r>0)$ and $f_{1-}=f_1(v_r<0)$, equation (2) reduces to:

$$\frac{\partial f_{1\pm}}{\partial r}+\frac{1}{|v_r|}\left(\frac{v_\varphi^2}{r}+\frac{e}{m_e}\frac{\partial\Phi(r)}{\partial r}\right)\frac{\partial f_{1\pm}}{\partial|v_r|}-\frac{v_\varphi}{r}\frac{\partial f_{1\pm}}{\partial v_\varphi}\pm\frac{\nu_{e-n}-i\omega}{|v_r|}f_{1\pm}=\pm\frac{eE_\varphi(r)}{m_e|v_r|}\frac{\partial f_0}{\partial v_\varphi} \qquad (3)$$

with solutions expressing the conservation laws for the total energy $\varepsilon=[m(v_r{}^2+v_\varphi^2)/2]-e\Phi=\text{const.}$ and for the angular momentum $M=m_e r v_\varphi=\text{const.}$ of the electrons, and describing the radial dependence of their distribution

$$f_{1\pm}(r)=\pm\int_{r_1}^{r}\frac{eE_\varphi(r')}{m_e|v_r(r')|}\frac{\partial f_0(r')}{\partial v_\varphi}\mathrm{e}^{\pm\alpha(r',r)}\mathrm{d}r'+\frac{e^{\mp\alpha(r,r_1)}}{\sinh\alpha(r_2,r_1)}\int_{r_1}^{r_2}\frac{eE_\varphi(r')}{m_e|v_r(r')|}\frac{\partial f_0(r')}{\partial v_\varphi}\cosh\alpha(r',r_2)\mathrm{d}r'. \qquad (4)$$

The latter is obtained by using the condition for continuity of f_1 at $v_r=0$: $f_{1+}(r=r_{1,2})=f_{1-}(r=r_{1,2})$. This condition is applied at $r=r_1$, the position of specular reflection of the electrons from the potential barrier near the discharge walls, and at $r=r_2$, the position in the middle of electron path between two consecutive reflections. The quantities denoted by α are integrals of the type of $\alpha(r_1,r_2)=\int_{r_2}^{r_1}\frac{\nu_{e-n}-i\omega}{|v_r(r')|}dr'$, in fact, related to the ratio of ω to the frequency characterizing the bounce motion of the electrons in the potential well.

The current density is

$$j_\varphi(r)=-e\int_{(\mathrm{v})}v_\varphi(f_{1+}+f_{1-})\mathrm{d}^3\mathrm{v}, \qquad (5)$$

where $\mathrm{d}^3\mathrm{v}=\mathrm{d}v_r\mathrm{d}v_\varphi\mathrm{d}v_z$. Polar co-ordinates $v_r=v_\perp\cos\phi,\ v_\varphi=v_\perp\sin\phi$ are introduced and the positions r_1 and r_2 are determined as roots of $|v_r(r')|=0$ with

$$|v_r(r')|=\sqrt{v_\perp^2\left(1-\frac{r^2}{r'^2}\sin^2\phi\right)+\frac{2e}{m_e}(\Phi(r')-\Phi(r))} \qquad (6)$$

obtained from the conservation laws. Integration over v_z of the Maxwellian distribution f_0 and change of the integration order in equation (5) yield the final analytical form of the current density:

$$j_\varphi(r)=\int_0^r G_1(r,r')E_\varphi(r')\mathrm{d}r'+\int_r^R G_2(r,r')E_\varphi(r')\mathrm{d}r' \qquad (7)$$

where

$$G_1(r,r') = \int_0^{\arcsin(r'/r)} \int_0^{\infty} Z(r,r',v_\perp,\phi)\mathrm{d}v_\perp \mathrm{d}\phi + \int_{\arcsin(r'/r)}^{\pi/2} \int_0^{|v_{\perp 1}|} Z(r,r',v_\perp,\phi)\mathrm{d}v_\perp \mathrm{d}\phi \ ,$$

$$Z = \frac{2e^2 m_e}{\pi(\kappa T_e)^2} \frac{r v_\perp^3 \sin^2\phi}{r' | v_r(r') |} n_e(r') e^{-\frac{m_e v_\perp^2}{2\kappa T_e}} \left(\sinh\alpha(r',r) + \frac{\cosh\alpha(r,r_1)}{\sinh\alpha(r_2,r_1)} \cosh(r',r) \right),$$

$$G_2(r,r') = \int_0^{\pi/2} \int_{|v_{\perp 1}|}^{\infty} \frac{2e^2 m_e}{\pi(\kappa T_e)^2} \frac{r v_\perp^3 \sin^2\phi}{r' | v_r(r') |} n_e(r') e^{-\frac{m_e v_\perp^2}{2\kappa T_e}} \frac{\cosh\alpha(r,r_1)}{\sinh\alpha(r_2,r_1)} \cosh\alpha(r',r)\mathrm{d}v_\perp \mathrm{d}\phi;$$

$$| v_{\perp 1} | = \sqrt{\frac{2e}{m_e} \frac{\Phi(r) - \Phi(r')}{[1-(r^2/r'^2)\sin^2\phi]}}.$$

In (7), G_1 and G_2 are the conductivity kernels and T_e is the electron temperature. The integral form of the current density accounts for the nonlocality: The current density at a given radial position is a function of the field along the entire trajectories of the electrons. The nonlocal conductivity is an effect of the thermal motion which transfers to a given r-position information about the rf field at all previous positions of the electron along its trajectory.

Equations (1) and (7) coupled in a set result in the spatial distribution of the high-frequency electric field and the current density in the discharge. The boundary conditions for $E_\varphi(r)$ are $E_\varphi(r=0)=0$ and $E_\varphi(r=R)=E_{\varphi 0}$ where $E_{\varphi 0}$ is adjusted to match the target total power P. In the numerics, the finite difference method is applied.

3. Results and discussions

Anomalous skin and nonlocality govern, respectively, the rf field penetration and the current-density formation under the conditions of the free-fall regime of maintenance of cylindrical inductive discharges. In fact, the analysis in the previous section completes the electrodynamical part of the model of such a discharge. In the presentation of the numerical results here, it is coupled with the fluid-plasma model description [11] of hydrogen discharges. The gas-discharge conditions are, as follows: gas pressure $p = 7\ \mathrm{mTorr}$, $R = 2.25\ \mathrm{cm}$ and power $P = 196\ \mathrm{W/cm}$ applied per an 1 cm length of the discharge. The plasma parameters [11] are: $T_e = 9.45\ \mathrm{eV}$, $\nu_{e-n} = 1.1\times10^7\ \mathrm{s}^{-1}$ and $n_e(r)$ and $\Phi(r)$ as given in figure 2. The frequency $f = \omega / 2\pi$ of the applied rf power is varied.

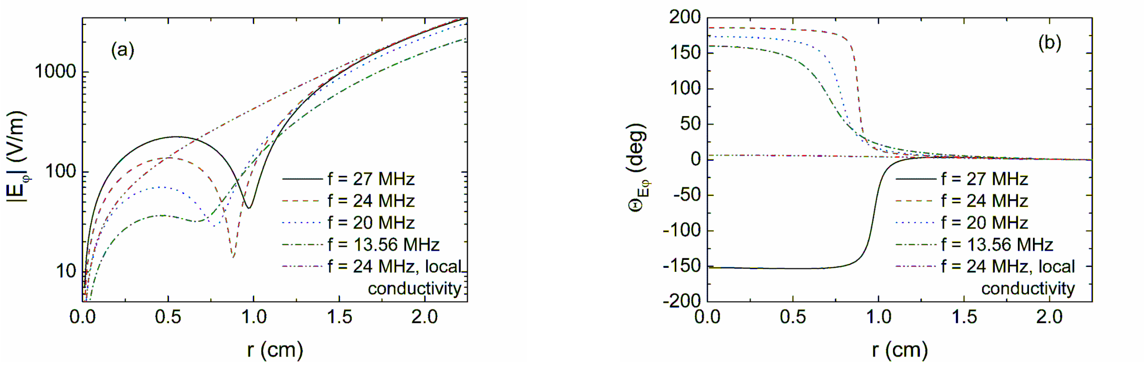

Figure 3. Radial distribution of the amplitude (a) and of the phase (b) of the rf electric field.

Figure 3 presents the results for the radial profiles of the amplitude and the phase of the rf electric field sustaining the discharge. In order to stress that local approach is not applicable to the given gas-discharge conditions, the field amplitude and phase obtained by using the expression for the local conductivity $(\sigma = (n_e e^2)/(m_e(\nu_{e-n} - i\omega)))$ are also shown. Nonmonotonic variations of the wave amplitude and phase across the radius resulting from the nonlocal electrodynamics replace the monotonic variations predicted by the local approach. Two regions of different types of radial variation of the amplitude show evidence: close to an exponential decay of the amplitude near to the discharge walls and a hump in the plasma interior. A slight linear variation of the phase accompanies the changes of the field amplitude in these two regions. However, the phase jumps sharply at the position of the drop of the field amplitude between the two regions. The deep extension of the field towards the discharge axis is due to the transfer – by the thermal electron velocity – of a directed-motion momentum gained in the skin layer: The electrons, reflected by the dc potential well back to the plasma interior, keep "memory" – through their momentum – about the rf field in the skin layer.

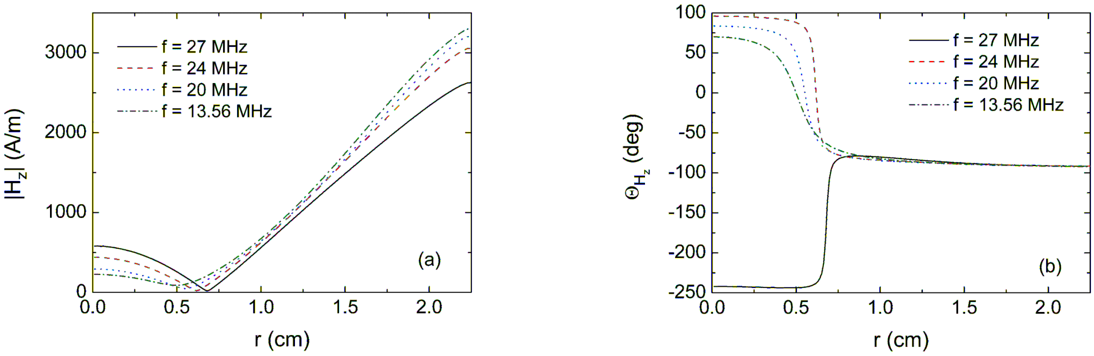

Figure 4. The same as in figure 3, but for the rf magnetic field.

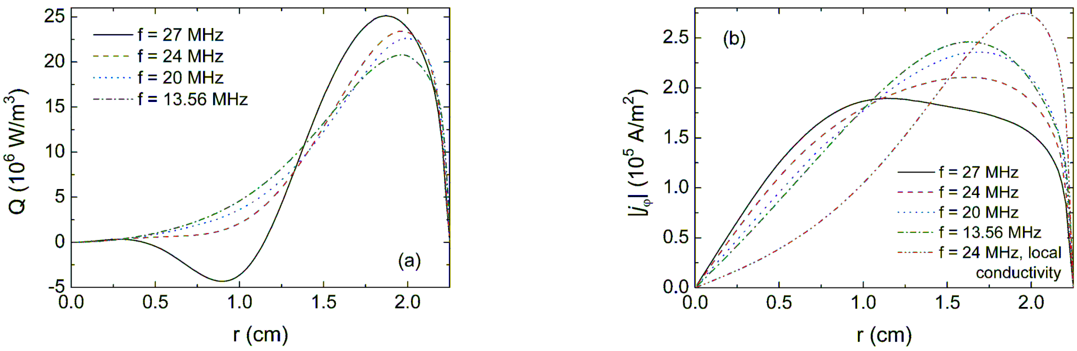

Figure 5. Radial distribution of the rf power deposition (a) and of the rf current density (b).

Since the gas pressure is low and, respectively, the mean free path of the electrons is large, a big difference could appear between the phase of the local field and that of the electron momentum, thus, strongly affecting the field and the current density in the plasma interior. Field variation close to the walls similar to that predicted by the local approach does not contradict to such a concept because the electrons coming from the plasma interior transfer relatively small momentum to the skin layer, compared to that gained in the field there and, by that reason, they could not affect significantly the current and the field in the skin layer. Connected to the E_φ-field according to $H_z(r) = (-\mathrm{i}/\omega\mu_0 r)[\mathrm{d}(rE_\varphi(r)/\mathrm{d}r]$, the magnetic field (figure 4) is also with nonmonotonic radial changes and a jump of the phase. The spatial distribution of the amplitude and the phase obtained in cylindrical discharges is in the trends of that discussed [4] in discharges with planar configuration, confirming that the nonmonotonic changes of the amplitude of the field and the jump of its phase outline general behaviour within the nonlocal electrodynamics. The frequency dependence of the effects can be related to the presence of a "second wall" in the discharge which provides conditions for a resonance of the wave field with the bounce motion of the electrons across the total cross section of the discharge. However, in cylindrical discharges the scenario of this transit time resonance is quite more complicated compared to discharges with a planar configuration [5].

The radial profile of the power deposition into the discharge (figure 5(a)) also shows effects specific for the nonlocal electrodynamics. The most drastic one is the negative power absorption obtained for the highest frequency (f = 27 MHz) shown in the figure: There is a spatial region in the discharge where the electrons, instead of taking energy from the field, give back energy to the wave. This effect of a reverse Landau damping is also a display of the importance of the thermal motion of the electrons. The negative power absorption results from an accumulation – due to the time variation of the field during the electron walk from the skin layer to the plasma interior – of a phase difference larger then $\pi/2$ between the momentum of the electron mainly gain in the skin layer, and the local field in the plasma interior. Figure 5(b) which shows the radial profiles of the current density amplitude confirms the discussions on the extension of the current channel into the plasma interior. The profile calculated by using the expression for the local conductivity, given for comparison, is concentrated near the walls.

4. Conclusions

Nonlocal electrodynamics of cylindrical inductive discharges sustained at low-gas pressure is developed. The difficulties in treating discharges with cylindrical configuration are due to having the azimuthal velocity as a variable which enters the conservation laws of the electron motion. The current density of the electrons is derived and coupled with the wave-field equation resulting in the spatial distribution of the field components of the transverse high-frequency waves which sustains the discharge as well as for the current density and the power deposition in the discharge. In a way, the study presents the electrodynamical part of the models of the free-fall-regime maintenance of cylindrical inductive discharges.

Deep extension of the wave field and of the current density into the plasma interior, jump of the phase of the field components as well as appearance of spatial regions of negative power deposition in the discharge outline the behaviour of the discharges sustained under conditions of low gas pressures and bring into evidence nonlocality associated with the thermal motion of the electrons.

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References

[1] Lieberman M A and Lichtenberg A J 1994 *Principles of Plasma Discharges and Materials Processing* (New York: Wiley)
[2] Alexandrov A F, Bogdankevich L S and Rukhadze A A 1988 *Foundations of Plasma Electrodynamics* (Moscow: Vyshaya Shkola), in Russian
[3] Kolobov V I and Economou D J 1997 *Plasma Sources Sci. Technol.* **6** R1
[4] Godyak V 2003 *Plasma Phys. Control. Fusion* **45** A399
[5] Kaganovich I D and Polomarov O 2003 *Phys. Rev. E* **68** 026411(1–12)
[6] Turner M M 1993 *Phys. Rev. Lett.* **71** 1844
[7] Weibel E S 1967 *Phys. Fluids* **10** 741
[8] Liberman M A, Meyerovich B E and Pitaevskiy L P 1972 *ZhETF* **62** 1737 (in Russian)
[9] Blevin H A, Greene J M, Jolly D L and Storer R G (1973) *J. Plasma Phys.* **10** 337
[10] Sayasov Yu S 1979 *Helv. Phys. Acta* **52** 288
[11] Paunska Ts, Schlüter H, Shivarova A and Tarnev Kh 2006 *Phys. Plasmas* **13** 023504 (1–10)