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# Simulation of a quantum phase transition of polaritons with trapped ions

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We present a system for the simulation of quantum phase transitions of collective internal qubit and phononic states with a linear crystal of trapped ions. The laser-ion interaction creates an energy gap in the excitation spectrum, which induces an effective phonon-phonon repulsion and a Jaynes-Cummings-Hubbard interaction. This system shows features equivalent to phase transitions of polaritons in coupled cavity arrays. Trapped ions allow for easy tuning of the hopping frequency by adjusting the axial trapping frequency and the phonon-phonon repulsion via laser detuning and intensity. We propose an experimental protocol to access all observables of the system, which allows one to obtain signatures of the quantum phase transitions even with a small number of ions.

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Trapped ions are among the most promising physical systems for implementing quantum computation [1] and quantum simulation [2]. Long coherence times and individual addressing allow for the experimental implementation of quantum gates and quantum computing protocols such as the Deutsch-Josza algorithm, teleportation, quantum error correction, quantum Fourier transformation, and Grover search [3]. Quantum simulation could be performed in the future by large-scale quantum computation [4]. With the currently available technology, tailored Hamiltonians can be modeled with trapped ions to simulate mesoscopic Bose-Hubbard systems [5], spin-boson systems [6], and spin systems [7]. Recently, a quantum magnet consisting of two spins has been successfully simulated experimentally with two trapped ions [8].

In this Rapid Communication, we propose a physical implementation of the Jaynes-Cummings-Hubbard (JCH) model using trapped ions. The JCH model was proposed in the context of an array of coupled cavities, each containing a single two-state atom and a photon [9]. Such a system is described by the combination of two well-known physical models: the Hubbard model [10,11], which describes the interaction and hopping of bosons in a lattice, and the Jaynes-Cummings model, which describes the interaction of an atom with a quantum field. The JCH model predicts a quantum phase transition of polaritons, which are collective photonic and atomic excitations. We shall show that the laser-driven ion chain in a linear Paul trap is described by a JCH Hamiltonian, wherein the ions and the phonons correspond to the atoms and the photons, respectively, in a coupled cavity array (Fig. 1). As in [5], the position-dependent energy and the nonlocal hopping frequency of the phonons are controlled by the trapping frequencies, while the effective on-site repulsion is provided by the interaction of the phonons with the internal states of the ions and can be adjusted by the parameters of an external laser field, namely, the Rabi frequency and the detuning. This on-site interaction is analogous to the photon blockade (photon-photon repulsion), where the strong atom-cavity coupling prevents the entering of additional photons into the optical cavity [12]. We shall show that many-body effects appear as a quantum phase transition between a localized Mott insulator (MI) and a delocalized superfluid (SF) state of the composite phononic and internal (qubit) states of the ions. Due to the collective nature of the excitations we distinguish between collective qubit and phononic SF and MI phases and the pure phononic SF phase, similar to the effects predicted in [13] for coupled cavity arrays.

Consider a chain of $N$ ions confined in a linear Paul trap along the $z$ axis with trap frequencies $\omega_q$ $(q=x,y,z)$, where the radial trap frequencies are much larger than the axial trap frequency $(\omega_{x,y} \gg \omega_z)$, so that the ions are arranged in a linear configuration and occupy equilibrium positions $z_i^0$ along the $z$ axis. Making a Taylor expansion around the equilibrium po-

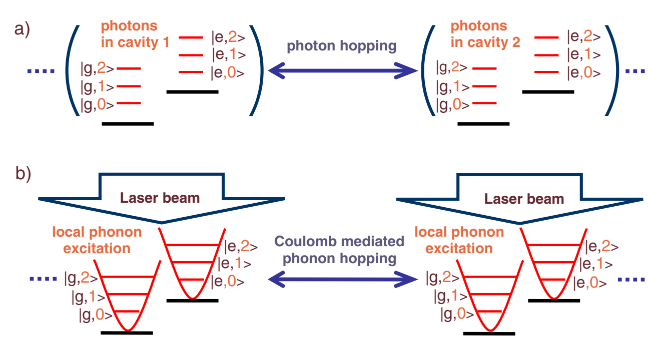


FIG. 1. (Color online) Analogy between phase transitions in coupled cavities and trapped ions: (a) coupled cavities each containing photons and single two-state atoms. Intercavity hopping is provided by an optical fiber. The strong coupling between the atoms and the photons leads to a photon-photon repulsion. (b) All ions are simultaneously interacting with a traveling wave in the radial direction. The laser-ion interaction creates an effective on-site interaction between the local phonons. The phonon hopping appears due to the Coulomb interaction and can be adjusted by the mutual distance of the ions.

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sition and neglecting $x^3$, $y^3$, $zx^2$, $zy^2$ and higher-order terms, the Hamiltonian in the radial direction $x$ reads [14]

$$\hat{H}_x = \frac{1}{2M}\sum_{k=1}^{N} \hat{p}_k^2 + \frac{M\omega_x^2}{2}\sum_{k=1}^{N} \hat{x}_k^2 - \frac{M\omega_z^2}{2}\sum_{\substack{k,m=1\\k>m}}^{N} \frac{(\hat{x}_k-\hat{x}_m)^2}{|u_k-u_m|^3}. \tag{1}$$

Here $\hat{p}_k$ is the momentum operator, $M$ is the ion mass, and $\hat{x}_k$ is the position operator of the $k$th ion about its dimensionless equilibrium position $u_k$. In Hamiltonian (1) the motion in the radial direction is decoupled from the axial motion. In terms of the normal modes $\omega_p$, Hamiltonian (1) reads $\hat{H}_x = \hbar\Sigma_{p=1}^{N}\, \omega_p(\hat{\tilde{a}}_p^\dagger \hat{\tilde{a}}_p + \frac{1}{2})$. Here $\hat{\tilde{a}}_p^\dagger$ and $\hat{\tilde{a}}_p$ are the phonon creation and annihilation operators of the $p$th *collective* phonon mode. However, if $\hat{x}_k$ and $\hat{p}_k$ are written in terms of *local* creation $\hat{a}_k^\dagger$ and annihilation $\hat{a}_k$ phonon operators, so that $\hat{x}_k = \sqrt{\hbar/2M\omega_x}(\hat{a}_k^\dagger + \hat{a}_k)$ and $\hat{p}_k = i\sqrt{\hbar M\omega_x/2}(\hat{a}_k^\dagger - \hat{a}_k)$, Hamiltonian (1) reads

$$\hat{H}_x = \hbar\sum_{k=1}^{N}(\omega_x+\omega_k)\hat{a}_k^\dagger\hat{a}_k + \hbar\sum_{\substack{k,m=1\\k>m}}^{N} \kappa_{km}(\hat{a}_k^\dagger\hat{a}_m + \hat{a}_k\hat{a}_m^\dagger), \tag{2}$$

where we have neglected higher-order (energy nonconserving) terms. The phonons are trapped with a position-dependent frequency

$$\omega_k = -\frac{\alpha\omega_z}{2}\sum_{\substack{s=1\\s\neq k}}^{N} \frac{1}{|u_k-u_s|^3}, \tag{3}$$

where $\alpha = \omega_z/\omega_x$ ($\alpha \ll 1$), and they may hop between different ions, with nonlocal hopping strengths

$$\kappa_{km} = \frac{\alpha\omega_z}{2}\frac{1}{|u_k-u_m|^3} \tag{4}$$

derived from the long-range Coulomb interaction [5].

The collective and local creation and annihilation operators are connected by the Bogoliubov transformation,

$$\hat{\tilde{a}}_p^\dagger = \sum_{k=1}^{N} b_k^{(p)}(\hat{a}_k^\dagger \cosh\theta_p - \hat{a}_k \sinh\theta_p), \tag{5}$$

which preserves the commutation relation, $[\hat{a}_k, \hat{a}_m^\dagger] = \delta_{km}$. Here $\theta_p = -\frac{1}{4}\ln\gamma_p$, with $\gamma_p = 1+\alpha^2(1-\lambda_p)/2$ and $\lambda_p$ are the eigenvalues, with eigenvectors $\mathbf{b}^{(p)}$, of the matrix $A_{km} = \delta_{km} + 2\Sigma_{s=1,s\neq k}(\delta_{km}-\delta_{sm})/|u_k-u_s|^3$ [14]. Using Eq. (5) one finds that the $p$th collective phonon state with zero phonons $|\tilde{0}_p\rangle$ is a product of $N$ local squeezed states, $|\tilde{0}_p\rangle = |\zeta_1^{(p)}\rangle, \ldots, |\zeta_N^{(p)}\rangle$, where

$$|\zeta_k^{(p)}\rangle = \sum_{n_k=0}^{\infty}\sqrt{\frac{(2n_k-1)!!}{(2n_k)!!}}\frac{(\tanh\theta_p)^{n_k}}{\sqrt{\cosh\theta_p}}|2n_k\rangle. \tag{6}$$

Here $|2n_k\rangle$ ($k=1,2,\ldots,N$) is the local Fock state with $2n_k$ phonons. For the center-of-mass phonon mode we have $p=1$ and $\cosh\theta_p = 1$; hence the collective ground state $|\tilde{0}_1\rangle$ is a product of local ground states, $|\tilde{0}_1\rangle = |0_1\rangle, \ldots, |0_N\rangle$. For a sufficiently small number of ions, we have $\cosh\theta_p \approx 1$ and $|\zeta_k^{(p)}\rangle \approx |0_k\rangle$. Since the lowest-energy collective vibrational mode in the radial direction is the highest mode $p=N$, we find that the SF ground state of Hamiltonian (2) is

$$|\Psi_{\mathrm{SF}}\rangle = \frac{1}{\sqrt{N!}}\left(\sum_{k=1}^{N} b_k^{(N)}\hat{a}_k^\dagger\right)^N |0_1\rangle|0_2\rangle\ldots|0_N\rangle. \tag{7}$$

Here we have assumed the commensurate case where the number of ions is equal to the number of phonons. We find that the ratio between the average number of local phonons in the ground state is given by the square of the oscillation amplitudes: $\langle\hat{n}_k\rangle/\langle\hat{n}_m\rangle = (b_k^{(N)}/b_m^{(N)})^2$, where $\hat{n}_k = \hat{a}_k^\dagger\hat{a}_k$ is the local phonon number operator.

We shall show that the laser-ion interaction induces an effective repulsion between the local phonons. This interaction provides the phase transition from phononic SF state to composite SF and MI phases of the joint phononic and qubit excitations. Consider ion qubits with a transition frequency $\omega_0$, which interact along the radial direction with a common traveling-wave laser light addressing the whole ion chain with frequency $\omega_{\mathrm{L}}$. The Hamiltonian of the system after the optical rotating-wave approximation is given by [15]

$$\hat{H} = \hat{H}_x + \hbar\Omega\left[\sum_{k=1}^{N}\hat{\sigma}_k^+ e^{i\eta(\hat{a}_k^\dagger+\hat{a}_k)-i\delta t} + \mathrm{H.c.}\right]. \tag{8}$$

Here $\hat{\sigma}_k^+ = |e_k\rangle\langle g_k|$ and $\hat{\sigma}_k^- = |g_k\rangle\langle e_k|$ are the spin flip operators, $|e_k\rangle$ and $|g_k\rangle$ are the qubit states of the $k$th ion, $\Omega$ is the real-valued Rabi frequency, $\delta = \omega_{\mathrm{L}} - \omega_0$ is the laser detuning, and $\eta = |\mathbf{k}|x_0$ is the Lamb-Dicke parameter, where $\mathbf{k}$ is the laser wave vector and $x_0 = \sqrt{\hbar/2M\omega_x}$ is the spread of the ground-state wave function. The Hamiltonian, after transforming into the interaction picture by the unitary transformation $\hat{U} = e^{i\hat{H}_0 t/\hbar}$, with $\hat{H}_0 = -\hbar\omega_x\Sigma_{k=1}^{N}\,\hat{a}_k^\dagger\hat{a}_k + \hbar\Delta\Sigma_{k=1}^{N}|e_k\rangle\langle e_k|$, in the Lamb-Dicke limit and after the vibrational rotating-wave approximation, reads

$$\hat{H}_I = \hbar\sum_{k=1}^{N}\omega_k\hat{a}_k^\dagger\hat{a}_k + \hbar\Delta\sum_{k=1}^{N}|e_k\rangle\langle e_k| + \hbar g\sum_{k=1}^{N}(\hat{\sigma}_k^+\hat{a}_k + \hat{\sigma}_k^-\hat{a}_k^\dagger) + \hbar\sum_{\substack{k,m=1\\k>m}}^{N}\kappa_{km}(\hat{a}_k^\dagger\hat{a}_m + \hat{a}_k\hat{a}_m^\dagger), \tag{9}$$

where $\hat{H}_I = \hat{U}^\dagger\hat{H}\hat{U} - i\hbar\hat{U}^\dagger\partial_t\hat{U}$. We assume that the laser is tuned near the red motional sideband $\delta = -\omega_x - \Delta$, with a small detuning $\Delta$ ($\Delta \ll \omega_x$). The coupling between the internal qubit and local phonon states is $g = \eta\Omega$. Hamiltonian (9) is valid when $\kappa_{km}, g \ll \omega_x$, which ensures that higher terms, which violate the conservation of the total number of excitations, can be neglected. The first three terms in Eq. (9) describe the Jaynes-Cummings model. The first two terms correspond to the energies of the local phonons and the ions, while the third term describes the laser-ion interaction. The fourth term in Eq. (9) describes the nonlocal hopping of phonons between different ions and allows the comparison to Hubbard systems. Hamiltonian (9) commutes with the total excitation operator $\hat{N}=\Sigma_{k=1}^{N}\hat{N}_k$, hence the total number of excitations is conserved. Here $\hat{N}_k=\hat{a}_k^\dagger\hat{a}_k+|e_k\rangle\langle e_k|$ is the number operator of the total qubit and phononic excitations at the $k$th site. If the laser detuning $\delta$ is tuned near the blue motional sideband then the system is represented by the anti-Jaynes-Cummings dynamics, which shows equivalent behavior by redefinition of the excitation operator $\hat{N}_k=\hat{a}_k^\dagger\hat{a}_k+|g_k\rangle\langle g_k|$. In the following we only assume small detunings $\Delta$ around the red-sideband transition, so that the anti-Jaynes-Cummings interaction does not occur.

In the JCH model the effective on-site interaction is provided by the interaction of phonons and qubit states at each site. The strength of the on-site interaction depends on external parameters, such as the Rabi frequency $\Omega$ and the laser detuning $\Delta$. This interaction creates an energy gap, which prevents the absorption of additional phonons by each ion. When the hopping frequency is increased the energy gap decreases and a quantum phase transition occurs between the SF and MI phases [16].

We describe the quantum phase transition between the MI and SF states by the variance $\mathcal{D}N_k=(\langle\hat{N}_k^2\rangle-\langle\hat{N}_k\rangle^2)^{1/2}$ of the number operator $\hat{N}_k$ with respect to the ground state of Hamiltonian (9) for fixed number of excitations [13]. If the on-site interaction between the phonons dominates the hopping, the ground-state wave function is a product of local qubit and phononic states for each site with a fixed number of excitations. Hence in the MI state, the variance $\mathcal{D}N_k$ for any $k$ vanishes. When the hopping term dominates the on-site interaction, then the ground state consists of a superposition of qubit and phononic states with delocalized excitations over the entire chain. In this state the variance $\mathcal{D}N_k$ at each site (i.e., each ion) is nonzero.

Figure 2 (top) shows the variance $\mathcal{D}N_k$ $(k=1,2,3)$ for a chain of five ions with five collective excitations versus the laser detuning $\Delta$ for fixed hopping frequency $\kappa=\alpha\omega_z/2$ calculated by an exact diagonalization of Hamiltonian (9). Due to the symmetry of the trap with respect to the center it is not necessary to plot the phase diagrams for ions 4 and 5. For sufficiently large negative detuning $\Delta$, there exists an energy gap, which prevents the absorption of additional phonons. Hence, the system is in the MI phase, where the qubit and phononic excitations are localized. When the detuning $\Delta$ increases, the energy gap decreases and the system makes a phase transition to the SF phase. The phase transition is stronger for the ions near the center of the trap due to stronger Coulomb interaction and increased hopping strengths and weaker at the ends of the ion chain. The comparison between the variance at the different sites demonstrates two characteristic features. First, there is a range of detunings where the qubit and phononic excitations at ion 1 (end of the chain) are predominantly in a MI phase, whereas the other ions are in the SF phase. Second, there is a broad range of $\Delta$ along which the joint excitations at all ions are in the SF phase.

Figure 2 (bottom) shows the variance of the qubit excitations $\mathcal{D}N_{a,k}$ with $\hat{N}_{a,k}=|e_k\rangle\langle e_k|$ [13]. This allows us to distinguish the following phases: in the region of large negative detuning $\Delta$ the collective and the qubit variances are small, indicating that the system is in the qubit MI phase. Increasing the detuning, the collective variance stays small but the qubit variance increases, which shows that the system is indeed in a collective MI phase. Approaching $\Delta=0$ the system makes a phase transitions into the collective qubit and phononic SF phases as now both collective and qubit variance are large. Finally, for sufficiently large positive detuning the qubit variance decreases but the collective variance stays large, which shows that the system is in the phononic SF phase.

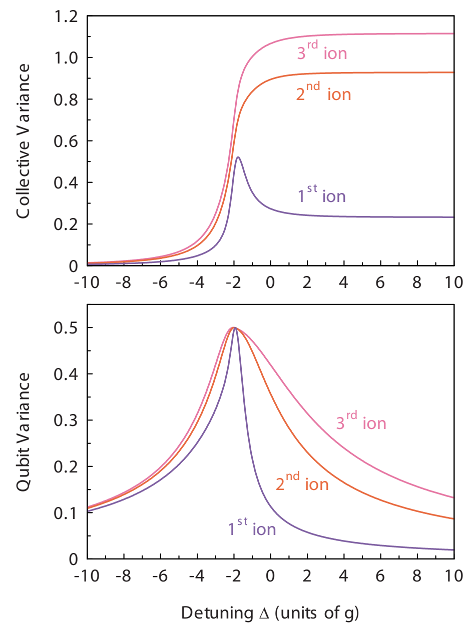


FIG. 2. (Color online) Total (qubit+phonon) variance $\mathcal{D}N_k$ (top) and the qubit variances $\mathcal{D}N_{a,k}$ $(k=1,2,3)$ (bottom) for a chain of five ions with five excitations as a function of the laser detuning $\Delta$ for fixed hopping $\kappa=0.3\ g$. Negative values of $\Delta$ correspond to blue detuning with respect to the red-sideband transition.

The experiment is started by initializing the ion chain to the superfluid state [see Eq. (7)] with $N$ phonons in the lowest-energy radial mode. To avoid off resonant excitation of unwanted radial modes, $\alpha$ could be increased temporarily so that $\omega_z/2\pi$ is increased to about 2 MHz. The lowest-energy radial mode is then separated by $2\pi\times0.5$ MHz from the other modes and is excited by a laser field on the blue sideband transition with more than 2 $\mu$s duration [17]. Then $\omega_z/2\pi$ is decreased adiabatically within 10 $\mu$s to $2\pi\times0.1$ MHz. This leads to a hopping frequency $\kappa/2\pi$ of 0.5 kHz. Now the coupling laser is switched on. The experimental proof for the phase transition can be carried out by local measurements on a single ion which should be performed faster than the hopping time $(\kappa/u^3)^{-1}$, where $u$ is the dimensionless average distance between two ions. In the case of five ions the hopping time amounts to 1 ms. For $^{40}Ca^+$ ions the qubit states could be represented by the ground state $S_{1/2}$ and the metastable state $D_{5/2}$. The laser, which creates the phonon-phonon repulsion, would be detuned to the red sideband of the quadrupole qubit transition between the two states. Then the readout could be performed by scattering photons on the dipole transition $S_{1/2}\rightarrow P_{1/2}$. This would lead to momentum recoil and changes of the phononic excitation, but to circumvent this we have to perform a measurement of the qubit states, which does not affect the phononic state as in the following steps. (1) Make a random guess for the qubit excitation. (2) If the guess was the $S_{1/2}$ ground state, then swap the population $S_{1/2} \Leftrightarrow D_{5/2}$ by a carrier $\pi$ pulse leaving the phononic excitation unchanged. (3) Now expose the ion to laser light on the dipole transition. (4) If the ion scatters light the guess was wrong and we have to discard the measurement and restart, otherwise the initial guess was right and we transfer the qubit excitation back to the $S_{1/2}$ ground state by another carrier $\pi$ pulse, then drive Rabi oscillations on the red sideband by perpendicular Raman light beams (difference momentum vector in the radial direction). (5) The phononic population can now be extracted by a Fourier analysis of the Rabi oscillations. In order to minimize the major error source of heating of the radial phonon modes the whole procedure for our choice of parameters must be performed faster than the heating time of the radial phonons of 2 s, obtained by $1/\omega^2$ scaling [18] of the experimental radial heating rates [19]. This leaves enough time for state preparation, evolution, and readout [20]. Fortunately the higher heating rates of the axial modes do not affect our scheme as they are decoupled from the radial modes.

In conclusion, we have proposed an implementation of the JCH model by trapped ions simulating polaritonic phase transitions in coupled cavity arrays. The system shows a MI to SF phase transition of the collective qubit and phononic excitations even with a small number of ions. The features can be easily measured by local addressing. Compared to atoms in optical cavities, our implementation is easier to manipulate, as all parameters can be tuned by changing the trap frequency, laser detuning, and intensity. Additionally, the system can be extended by adding impurities of ions with different masses, which allows for simpler addressing of the radial phonon modes and a separation of coexistent phases.

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