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PULSED ULTRASONIC IMAGING BY CONVOLUTION OF THE FOURIER TRANSFORMS OF TRANSMITTED THROUGH ELEMENTARY SUB-OBJECTS ACOUSTIC FIELDS

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Андреана Андреева, Марина Бурова. ИМПУЛСНО УЛТРАЗВУКОВО ИЗОБРАЗЯВАНЕ ЧРЕЗ КОНВОЛЮЦИЯ НА ФУРИЕ-ТРАНСФОРМАЦИИТЕ НА ПРЕМИНАЛИТЕ ПРЕЗ ПОДФОРМИТЕ, ИЗГРАЖДАЩИ ОБЕКТА, АКУСТИЧНИ ПОЛЕТА

Обект е осветен с къси акустични импулси. Амплитудите на преминалите през отделните елементи (прости подформи, изграждащи обекта) акустични импулси са регистрирани чрез сканиране с ултразвуков преобразувател. Преминалите през отделните елементи акустични импулси са разделени във времето поради разликата в изминатия път. Фурие-трансформация върху поредицата от цифрови стойностти за всяко амплитудно поле (преминалите акустични импулси през съответната подформа) е изпълнена чрез компютърна програма. Фурие-трансформациите представляват Фурие-образи на отделните прости форми. Профилът на надлъжното сечение на изследвания обект е реконструиран въз основа на теоремата за конволюцията. Образът е получен чрез обратната трансформация на конволюцията на отделните Фурие-трансформации. Показано е, че реконструкцията не зависи от броя на простите подформи, изграждащи обекта.

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An object is illuminated with short acoustic pulses. The amplitudes of the transmitted through the separated elements (simple forms, constructing the object) acoustic pulses are reg-

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istered by a scanning with ultrasonic transducer. The transmitted acoustic pulses through the simple forms are separated in time because of the path difference. The Fourier transformation of the digital data of each amplitude field (the transmitted acoustic pulses through the corresponding form) is performed by a computer program. The Fourier transformations represent Fourier images of the separated simple forms. The profile of the object longitudinal section is reconstructed on the basis of the convolution. The image profile is obtained from the reverse transformation of the convolution of the Fourier transformations. It is shown that image reconstruction is independent of the number of the simple forms.

Keywords: Fourier transformation, convolution, deconvolution, ultrasonic imaging, reverse transformation

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1. INTRODUCTION

The illuminating of an object by short acoustic pulses and the registration of the transmitted (diffracted) fields in digital form with following computer processing, is the base of the ultrasonic imaging. The different mathematical models are used for digital processing of the registered amplitude fields with purpose of object imaging.

One approach used in the ultrasound tomography for image reconstruction, is based on the Fourier transformation theorem of the object section. The theorem relates the Fourier transform of a projection to the Fourier transform of the object along a radial line. The Fourier transform of a parallel projection of an image $f(x,y)$ taken at angle θ gives a slice of the two-dimensional transform $F(u,v)$ (subtending an angle θ with the u-axis along) of the object a line rotated by θ. If an infinite number of projections are taken, then $F(u,v)$ would be known at all points in the uv-plane and the object function $f(x,y)$ can be recovered by using the inverse Fourier transform [1].

The other approach is based on the Abbe theory. According to this theory, the diffraction spots in the back focal plane act as point sources of secondary wavelets in the sense of Huygens. These arrive at the image plane and interfere constructively to create the image. Therefore a Fourier series of the studied object in spatial frequencies may be realized, if the each harmonic in the Fourier transformation (obtained by using a lens or by computer program) is multiplied by a term, giving angle distribution of the corresponding spatial frequency. The object image can be obtained after summing up in all spatial frequencies [2, 3]. The method is practically effective when the wavelength is of the same order of magnitude as the object dimensions. The reconstruction of the image depends on the discretization step of the interval at the summing up. The correct reconstruction is when the discretization step is equal to the step of sampling in the data plane.

Because the object is composed from elements with different thickness (a set of simple forms), a series of echo-signals is observed in the transmitted (diffracted) field. Each echo-signal corresponds to transmitting of acoustic pulse through some simple form.

The profile of each simple form can be reconstructed, using the mentioned above algorithm for processing of the registered amplitude field (a Fourier transformation of the data and summing up in all spatial frequencies – reverse transformation) [2, 3]. In [3] the reconstruction of one-dimensional profile of the longitudinal section of the object is obtained by an assembling of the both simple forms [3]. The assembling is more difficult process if the object is composed from more simple forms.

Fourier transformation must be applied for each data array, because the amplitude field of each pulse is registered in corresponding time domain for every scanning point. The Fourier image of the object will be "mapping" of the Fourier transformation of the separate simple forms. The "mapping" can be expressed mathematically as a convolution of the Fourier transformations of the separated forms.

The reverse operation deconvolution is used to improve the resolution of ultrasound images as the blurring caused by the ultrasonic system is removed [4]. Spiking deconvolution and blind deconvolution with different parameters is used to build inverse filters of the ultrasound pulse. Applying the inverse filters to the measured results in sharper signals that are used for image reconstruction [5].

A convolution procedure for improving the lateral resolution of ultrasonic images is proposed, as the starting point is the approximation of the sound-beam profile by model functions that arise from the convolution of rectangular functions of varying width [6].

In [7] the two-dimensional images of the local values of ultrasonic wave's propagation velocity in the phantom's internal structure (ultrasonic tomograms) are reconstructed by using the convolution and back-projection algorithm from the measurements of average values of ultrasonic signals' runtime propagated from many directions around the object dipped in water.

The reconstruction of the object image by a convolution of the Fourier transformations of the registered amplitude fields (transmitted through the object forms with different thickness) is the purpose of the present paper. The pulses passed through forms with different thickness are registered separately at each point of scanning. According to the convolution theorem and the use for prediction of the diffraction pattern from complicated grating, the Fourier object image is "mapping" of the Fourier transformations of the separate simple forms.

2. THEORY

One of the most important concepts in the Fourier theory is that of a convolution. Mathematically, a convolution is defined as the integral over all space of one function at x times another function at u-x. The integration is taken over the variable x (which may be a 1D or 3D variable), typically from minus infinity to infinity over all the dimensions. So the convolution is a function of a new variable u, as shown in the following equations. The cross in a circle is used to indicate the convolution operation

$$C(u) = f(x\otimes)g(x) = \int_{\text{space}} f(x)g(u-x)dx = \\ g(x)\otimes f(x) = \int_{\text{space}} g(x)f(u-x)dx \tag{1}$$

As it seems from eq. (1) it doesn't matter which function you take first, i.e. the convolution operation is commutative:

$$f(x) \otimes g(x) = g(x) \otimes f(x). \tag{2}$$

The convolution theorem has two statements:

1. The Fourier transform of the convolution of two functions is equal to the product of their separate Fourier transforms:

$$F(f \otimes g) = F(f)F(g). \tag{3}$$

2. The Fourier transform of the product of two functions is equal to the convolution of their separate Fourier transforms:

$$F(fg) = F(f) \otimes F(g). \tag{4}$$

In the analysis of the diffraction experiments, the convolution theorem is very useful. It allows diffraction patterns from complex objects to be explained in terms of the convolution and multiplication of simple object functions with simple diffraction patterns.

According to eq. (4), the Fourier transformation of the product (the object, composed from simple forms) is a convolution of the Fourier transformation of the functions, describing the simple objects. The convolution operator involves "mapping"of the function $F(g)$ into the function $F(f)$ or the function

$F(f)$ into $F(g)$. This is in the case, when the complex object is composed from two simple forms. When the object is composed from more forms, for example from three, it can be shown the Fourier transformation of the object image is the reverse transformation of the convolution of the Fourier transformation of the separate object forms. If the functions, describing the objects forms are f, g and w, using eq.(4) we may obtain the Fourier transformation F1 of the product of (fg).

$$F1(fg) = F(f) \otimes F(g). \tag{5}$$

If convolution is realized between the right part of the eq. (5) with Fourier transformation of the function w, i.e. F(w), the Fourier transformation of the product may be written as follow:

$$F2(fgw) = F(f) \otimes F(g) \otimes F(w). \tag{6}$$

Therefore the object image is the reverse transformation of the convolution of the Fourier transformations of the functions, describing of the simple forms, composing the object. The convolution operation in (6) is commutative, it does not matter the function sequence in the right part. The eq. (6) remains valid in the case when the object is composed from more simple forms.

The registered amplitude fields will present functions time-shifted each to other, when the acoustic pulse is passed though the object composed from simple forms with different thickness. The obtaining of object image is easy calculating Fourier transforms of the time-shifted functions and multiplication.

3. EXPERIMENTAL SET-UP

The experimental setup for registration of the amplitudes of transmitted fields is shown in Fig. 1. A longitudinal section and 3D visualization of the studied object are shown in Fig. 2 and Fig. 3 correspondingly.

The electric pulse passes from the amplifier by coaxial cable to the ultrasonic transducer (the diametrical size is 2 cm), which radiates an acoustic pulse. The transducer is located in a glass ripple tank, which is full of water. It radiates in the water acoustic pulse with compression polarization of the wave. A back flat surface of the object (composed from three simple forms) is set parallel to the radiated transducer. The ultrasonic pulse is incident on the object's surface at the normal. The ampli-

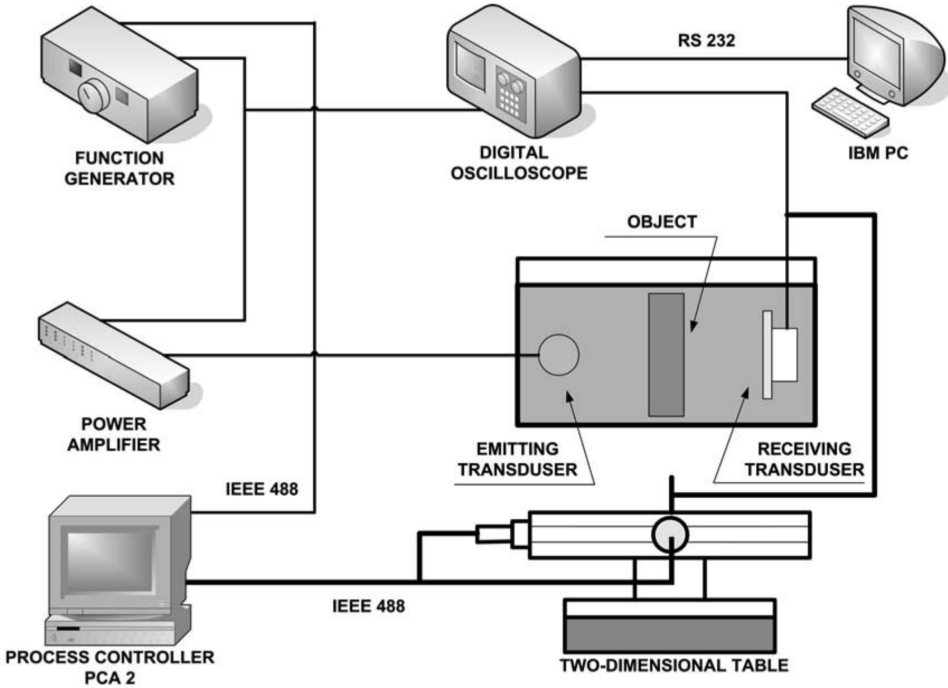

Fig. 1. Experimental set-up

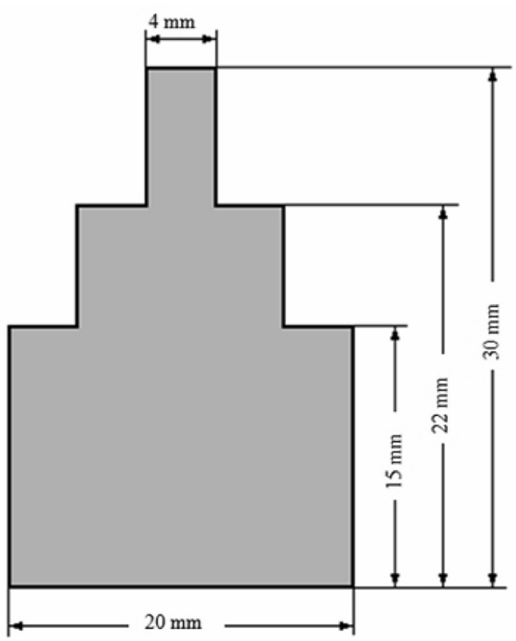

Fig. 2. The profile of the object longitudinal cross-section

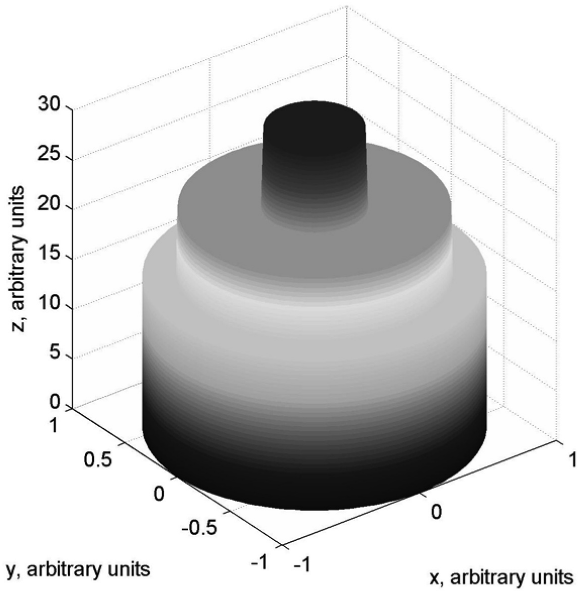

Fig. 3. 3D visualization of the studied object

tude of the transmitted signal is also registered by the receiving transducer. The registered pulses are observed on the digital oscilloscope Tektronix 11201. The signal is digitized (with 10240 point of sampling) and it is averaged with a digital filter to improve the ratio signal/noise. The receiving transducer is by the computer controlled two-dimensional coordinate table moved. The step of the movement of the transducer is 0.3 mm. A blend of 1 mm x 1 mm is placed on the receiving transducer in order to register the amplitude of the transmitted field at each step of moving. Three acoustic pulses separated in time domains are observed for every step of moving. The amplitude of every echo-signal in corresponding time domain is measured and saved for each point of scanning (or in each element of the acoustic array or matrix). The time domains (left and right limits for a measurement of the separate pulse) are defined automatically by MatLab program, which controlls the digital oscilloscope.

The pulse duration in every time domain (containing a transmitted pulse) is controlled by oscilloscope measurement function "delay". In this way a overlapping between the pulses may be avoided. By the interface RS232 each registered amplitude value is translated to PC. Three sets of data, i.e. three separate files of the amplitude values of the transmitted through the object components field are obtained.

The transmitted amplitude acoustic fields are registered with 130 points at the scaning step 0.3 mm and at distances between the object and the data plane z = 60 mm.

In accordance with Nyquist's sampling criterion, for distinct images, it must sample the wave front at least twice in the shortest spatial wavelength in the wave front. Since the shortest possible spatial wavelength in the aperture is λ, the sampling step $\lambda/2$ is adequate (or an array or matrix of elements spaced by $\lambda/2$). In our case wavelength λ is 1.3636 mm.

The Function generator type ROHDE & SCHWARZ AFG generates RF signal (sine burst) with the following parameters: frequency f = 1.1MHz, rise time t = 159.6 ns, fall time t = 168.2 ns, output level U = 1V, phase offset 0°, interval between the pulses T = 10 ms, number of the sinusoids N = 2. The continuance of the pulse is 1.82 μs.

The amplifier type ENI has the following characteristics: frequency range 150 kHz – 300 MHz, U_i^{max} = 1 V, P_{max} = 10 W.

4. DISCUSSION

The obtained in digital form amplitude fields transmitted through three separate simple forms are saved in three files and they are processed by MatLab function "smooth". Each amplitude field is sampled with 130 points. The amplitude field transmitted through the thickest segment is function "f" in eq. (6), the function "g" is the amplitude field transmitted through middle part and the function "w" through thinnest one. The Fourier transformation of the digital data for each file is performed by MatLab program. The realization of eq. (6) is made by MatLab convolution function. As it is mentioned above the convolution operation in (6) is commutative, it does not matter the function sequence in the right part. The inverse transformation of the convolution between convolution $F(f)$ and $F(g)$ is shown in Fig. 4.

The inverse transformation of the convolution between $F(f)$ and $F(w)$, and between $F(g)$ and $F(w)$ are shown in Fig. 5 and Fig. 6 correspondingly. The longitudinal section of the studied object is shown in Fig. 7 as inverse transformation of the convolution between $F(f)$, F(g) and $F(w)$. The reverse transformation is performed with 130 points. The 3D visualization of the object (obtained from the one-dimensional profile in Fig. 7 by spherical coordinates) is shown in Fig. 8. As it is seems from Fig. 7 the convolution function "smooth" the image additionally. For distinct image obtained by a convolution, the overlapping of the signals is not desirable. The presence of overlapping of the transmitted pulses will lead to distortion of the image. Especially at an im-

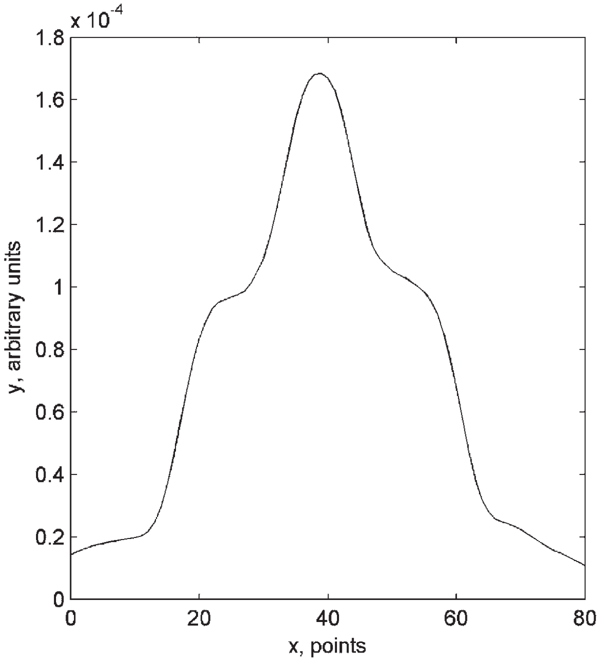

Fig. 4. The inverse transformation of the convolution between *F*(*f*) and *F*(*g*)

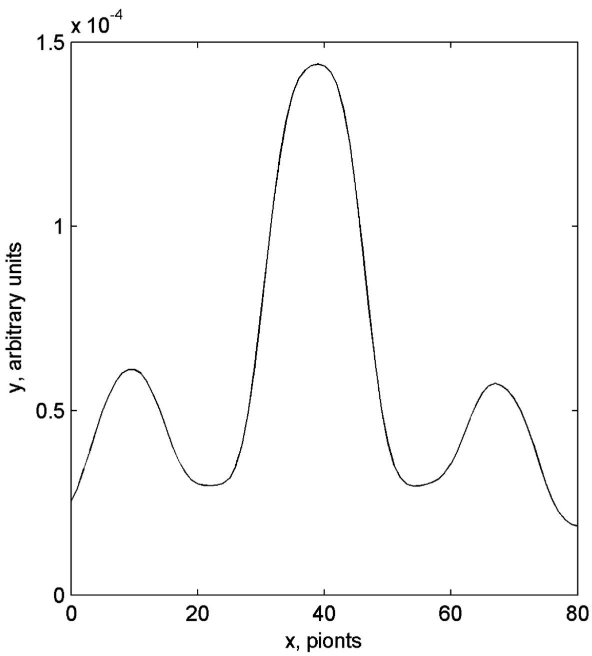

Fig. 5. The inverse transformation of the convolution between *F*(*f*) and *F*(*w*)

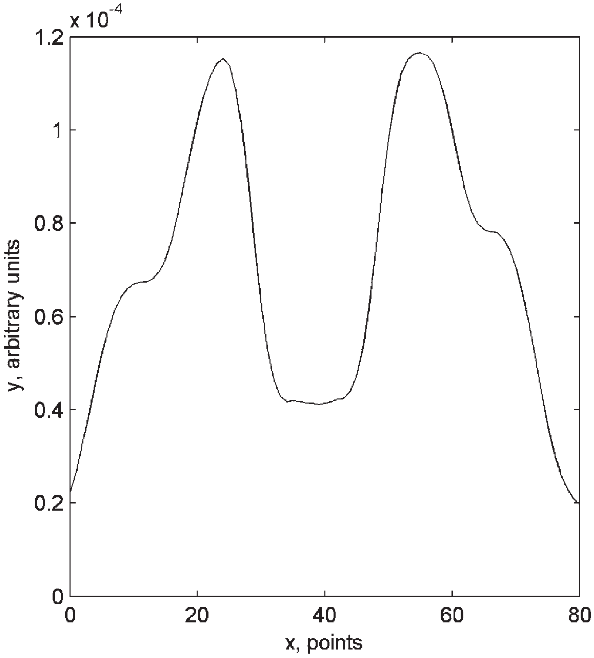

Fig. 6. The inverse transformation of the convolution between *F*(*g*) and *F*(*w*)

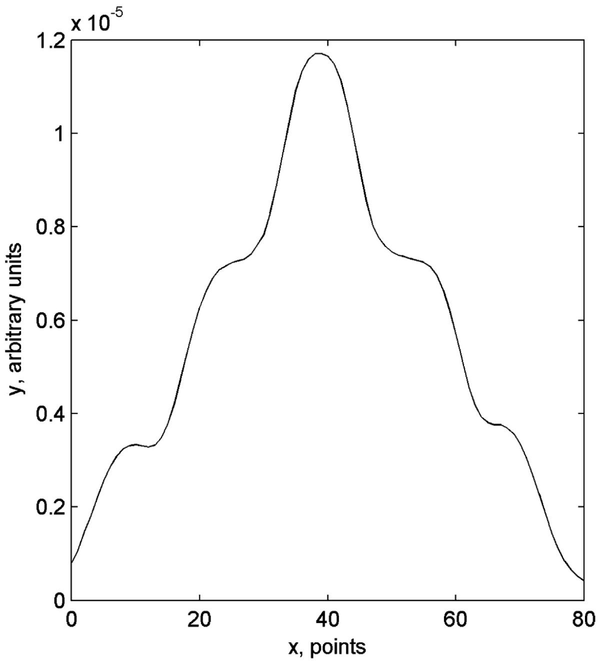

Fig. 7. The reconstructed profile of the object longitudinal section by the inverse transformation of the convolution between *F*(*f*), F(g) and *F*(*w*)

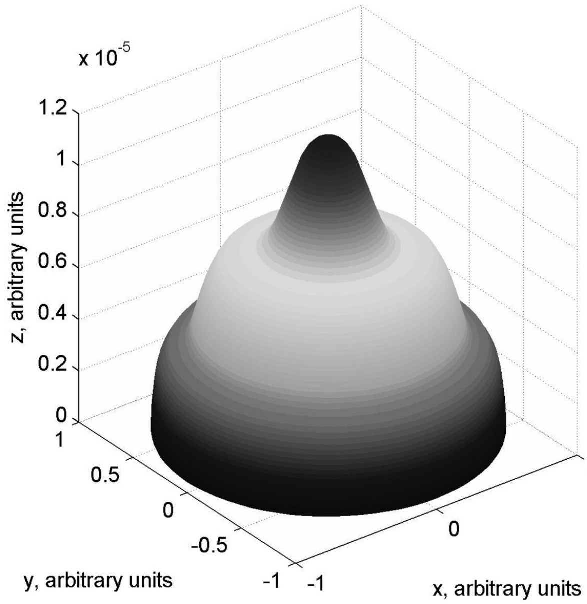

Fig. 8. 3D visualization of the object obtained from profile in Fig. 7

age of the objects, for which the sound velocity is high and the differences in the object dimensions are small. In this case the used acoustic pulse must be enough short, the data plane must be in the far field and to use liquids with low acoustic velocities.

5. CONCLUSSION

When the object composed from simple forms is illuminated with short acoustic pulses, it may be reconstructed using the second statement of the convolution theorem. In this case the Fourier transformation of the object is a convolution of the Fourier transformations of the transmitted through the simple forms acoustic fields. The convolution of the Fourier transformations of the fields is appropriate method for object imaging because of the fact that the transmitted acoustic pulses in the each point of scanning are time-shifted. If the transmitted field through the object is registered by acoustic matrix, the matrix convolution and three-dimensional visualization of the object may be realized easy.

REFERENCES

[1] Kak, A. C., M. Slaney. Principles of Computerized Tomographic Imaging. SIAM, 2001.
[2] Andreeva, A., M. Burova, J. Burov. *Bulg.J.Phys*., 2007, **34**, 59.
[3] Andreeva, A., M. Burova, J. Burov. *Ulltrasonics*, 2007, **46**, 219.
[4] Udantha, A., A. Petropulu, J. M. Reid. *Proc. SPIE*, 1994, **2296**, 191.
[5] Stotzka, R., N. V. Ruiter, T. O. Mueller, R. Liu, H. Gemmeke. *Proc. SPIE*, 2005, **5750**, 315.
[6] Kohl, W., R. Schwartz. *Ultrasonics*, 1981, **19**, 273.
[7] Opielinski, K. J., T. Gudba, P. A. Lewin, W. Sachse. *Ultrasonics International. Conference N°3*, Granada, ESPAGNE, 2004, **42**, 705.