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1-ARM PHASE OPTICAL BRIDGE WITH AXIAL SYMMETRY (WITH INTERFERENCE NEWTON FRINGES) FOR MEASURING SMALL PERIODIC DISPLACEMENTS

Indexing terms: Displacement measurement. Bridge instruments, Vibration measurement. Optical modulation, Phase modulation

A 1-arm phase optical bridge with axial symmetry (with interference Newton fringes) is developed for the measurement of small periodic displacements of acoustic frequency. Using the synchronous detection method, a vibration with an amplitude as small as 3×10^{-14} m is measured.

Various optical methods have been used for the noncontact measurement of small periodic displacements (1-6). In these, the vibrating object changes (modulates) the direction of propagation, the polarization, the phase of the frequency of an electromagnetic wave.

The phase-bridge method is distinguished by high sensitivity, high precision, and by a wide range of measurable periodic displacements. In it, the vibrating object modulates the phase of a monochromatic light beam. The latter interferes with another coherent monochromatic beam with a constant phase. The measurement of the phase modulation, respecting the amplitude of the vibration, is reduced to the measurement of a variable luminous flux.

In the measurement of small periodic displacements, 2-arm phase bridges of the type of the Michelson interferometer are most often used. In some works, the semitrans parent mirror of the Michelson interferometer is replaced by a Bragg cell to secure an additional amplitude modulation of the two interfering light beams.[3, 4]

Compared with 2-arm optical phase bridges, the use of an 1-arm phase optical bridge with axial symmetry offers certain

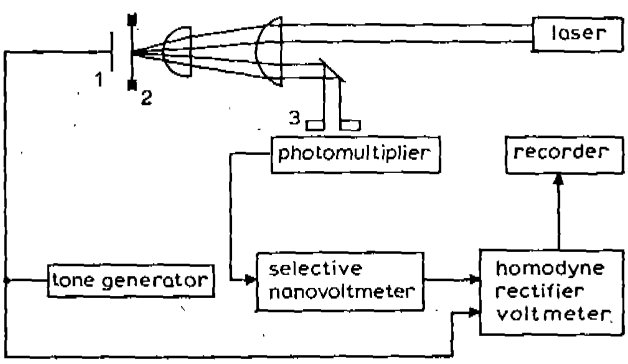

Fig. 1 ***Equipment***

advantages: the construction of the interferometer is substantially simplified, which secures an easy adjustment, and makes it possible to produce easily low-order interference; the intensity of the light beam is most fully used; the noise from the mechanical vibrations is strongly reduced, because the interferometer has only one short arm (one usually works with low-order interference).

Fig. 1 is a block-diagram of the equipment used for measuring small periodic displacements. The surface of the vibrating plate 1 (the vibrations of which are to be measured), and the convex surface of the planoconvex lens 2 with a small radius of curvature *R*, enclose an air layer. The latter is illuminated with monochromatic light from an He-Ne laser (light wavelengths λ = 6.328 x 10^{-7} m).

The luminous flux $\varphi(r)$ incident on the cathode of the photomultiplier as a function of the radius *r* of the blend 3 is

$$\Phi(r) = \Phi_{\sim}(r)\cos\frac{\pi}{\lambda}\left(\frac{r^2}{R} + 4d - \lambda\right)\sin\frac{\pi r^2}{\lambda R} + \Phi_{=}(r)$$

$$\Phi_{=}(r) = I_0 \pi r^2 \qquad \Phi_{\sim}(r) = I_0 R \lambda \tag{1}$$

provided that the two light beams reflected from the surfaces enclosing the air layer have the same intensity I_0, d being the layer thickness. The maximum modulation of the luminous flux, with respect to the current across the photomultiplier, i.e. maximal $|\partial f/\partial d|$ when d is varied, is obtained under the following conditions

$$d_0 = (n+1)\,\lambda/4 \qquad r_0 = \sqrt{\frac{\lambda R}{2}} \tag{2}$$

n being the order of the interference maximum at the central fringe. From eqn. 1 for the luminous flux incident on the photomultiplier cathode at $r^2 = \lambda R/2$, one obtains, substituting u, $u_{=}$ and $u_{\sim}$ the voltage across the load resistance of the photomultiplier for $\Phi(r), \Phi_{=}(r)$ and $\Phi_{\sim}(r)$,

$$u = u_{=} + u_{\sim}\sin\frac{4\pi}{\lambda}d \tag{3}$$

the u_s being proportional to the Φ_s as a result of the linearity of the lux illuminance/current ampere characteristic of the photomultiplier.

When plate 1 is displaced at a distance $\lambda/4$, the central interference maximum [at d_0 = $(n+1/4)\lambda/2$] is replaced by its neighbouring minimum [at d_0 = $(n+3/4)\lambda/2$]; consequently, the voltage across changes from a maximum value $u_{max} = u_{=} + u_{\sim}$ to a minimum value $u_{min} = u_{=} - u_{\sim}$. Hence, it follows that

$$u_{\sim} = \frac{u_{max} - u_{min}}{2} \tag{4}$$

Now, if plate 1 vibrates according to the sine law $d = d_0 + d'\sin\omega t$ ($d' \ll \lambda$), an alternating voltage of the same frequency and with an effective value

Δu_{ev} results across R_T. From eqn. 3, using eqn. 4, one obtains

$$d' = \frac{\sqrt{2}}{2}\frac{l}{p}\frac{\Delta u_{ev}}{u_{max}-u_{min}} \qquad \textbf{(5)}$$

provided that the conditions of eqns. 2 are observed.

The use of eqn. 5 in computing the amplitude of the vibration requires that the distance d_0 between the plate and the lens be preliminarily so adjusted that the luminous flux incident on the photomultiplier cathode be $(f_{max}+f_{min})/2$; i.e. that the voltage across R_T be $(u_{max}+u_{min})/2$.

The sensitivity of the method, i.e. the lowest amplitude that can be measured, is limited by the shot-noise level of the photomultiplier as well as by the indeterminacy principle.[7, 8] The shot noise of the photomultiplier hinders the measurement of amplitudes lower than d''_{min}, determined by the inequality

$$d''_{min} \ge \frac{l}{4pr_0^2 g}\sqrt{\frac{e\Delta f}{aI_0}} = \frac{l}{p(u_{max}-u_{min})}\sqrt{eu_0\Delta f R_T} \qquad \textbf{(6)}$$

where Δf is the bandwidth of the electronic equipment following the photomultiplier, e is the electronic charge, $g = \sin(\Delta w/c)d/(\Delta w/c)d$ is the factor of monochromatism, c is the velocity of light, a is the sensitivity of the photomultiplier and $u_0 = (u_{max}+u_{min})/2$ is the voltage across R_T when the vibrating plate is at its equilibrium position. Eqn. 6 is obtained by equating the average value of the voltage amplitude

$$\overline{\Delta u^2} = \frac{(\Delta u)^2}{2} = \frac{\{(\sqrt{2})\Delta u_{ev}\}^2}{2} = (\Delta u_{ev})^2 =$$
$$= 2\frac{d''_{min}}{l^2}p^2(u_{max}-u_{min})^2$$

to the average value of the photomultiplier noises, $u^2 = 2eu_0\Delta f R_T$ passing through the equipment. According to the indeterminacy principle, the size pr^2 *of* the light spot on the vibrating surface determines, in principle, the lowest possible amplitude d'''_{min} of the vibration that can

be measured by means of the inequality

$$d'''_{min} \ge \frac{m l^2}{8pr^2} \qquad \textbf{(7)}$$

where m is the order of the interference maximum.

On using synchronous detection at a frequency of vibration of the plate 991 Hz, $\Delta f = 0{,}01$ Hz, $u_{max} = 24$ V, $u_{min} = 9$ V, $R_T = 47$ kW, the smallest vibration that can safely be measured corresponds to a voltage of the order of $d' = 3\times10^{-6}$ V (Fig. 2). This corresponds to a vibration amplitude $d' = 3\times10^{-14}$ m. The latter amplitude is one order of magnitude higher than the lowest amplitude, measurable under the conditions of the experiment

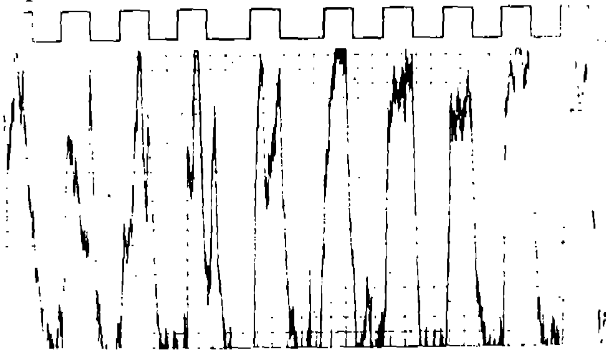

Fig. 2 ***Frequency of vibration***

Vertical scale: 1 -5 x 10-7 V /division Horizontal scale: 2 min /division

$(m = 2,\ r = 3\times10^{-3}\,m)$ in accordance with the indeterminacy principle (expr. 7) at the given noise of the electronic equipment (eqn. 6).

The equipment as described was used to measure vibrations of acoustic frequency (up to 10^4 Hz). It can be employed for measuring the electrostriction, magnetostriction and piezoelectric coefficients of various materials, as well as for investigating phase transitions in ferroelectrics and ferromagnetics. On cooling, a considerable enhancement of the sensitivity by 1-2 orders of magnitude can be expected, owing to the reduction of the shot noise of the photomultiplier, Such an enhancement may be important for the problem of detecting gravitational waves.[8, 9] The use of the above-described equipment in the measurement of small vibrations

of high frequency Ω_1 (e.g. in the case of surface waves) is possible by modulating the intensity of the laser beam with a frequency Ω_2 close to Ω_1 and measuring the amplitude of the photomultiplier signal at a frequency $\Omega_2 - \Omega_1$.

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M. I. BORISOV

J. I. BUROV

Department of Solid State Physics
Faculty of Physics
Sofia University
Blvd. A. Ivanov 5, Sofia 26, Bulgaria

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