15 August 1989 / Vol. 28, No. 15 / APPLIED OPTICS p. 3343

Multibeam interferometric methods for measuring very small periodic displacements

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A multibeam Pabry-Perot interferometer is used for measuring very small periodic displacements (SPD). The steepness of such an interferometer as well as the high gain of the photomultiplier and the powerful laser allow one to measure SPD of the order of 10^{-19} m at room temperature. Analysis of all the noise factors restricting the sensitivity (heat fluctuations of the surfaces of the mirrors, shot effect, radiation pressure noise) shows that at room temperature the heat noise is the most important.

1. Introduction

Nowadays measurements of small periodic displacements (SPD) are widely used in fundamental and practical problems such as gravitational wave detection,[1-3] noncontact measurement of the parameters of surface acoustic waves,[4,5] nondestructive testing,[6,7] photo-acoustic microscopy,[8] and photo-acoustic spectroscopy [9,10] .There are two limiting cases: either to measure SPD in a narrow frequency band as in gravitational wave measurements or to measure SPD in a wide frequency band as in nondestructive testing.

Different kinds of optic two-arm phase bridges of a Michelson type interferometer have been used in which the vibrating specimen serves as one of the interferometer's mirrors. Bernstein[11] measured SPD of the order of 2 x 10^{-12} m. Whitman et al.[12] replaced the semitransparent mirror of the Michelson interferometer with a Bragg cell for high frequency modulation of the light beams and the measured SPD was of the order of 2.07 x 10^{-12} m. A similar experimental setup was described by De La Rue et al.[5] The highest sensitivity was obtained by Etzold [4] (10^{-13} m) and by Forward [3] (1.6 x 10^{-14} m). A sensitivity of the order of 3 x 10^{-14} was obtained by Burov et al.[13,14] at low frequencies with the help of a one-arm optic phase bridge using Newton's fringes. Such a bridge allows a reduction of fluctuation noise caused by the mechanical displacements of the two arms of the interferometer.

Thomson et at.[15] at room temperature using a multi-beam Fabry-Perot interferometer reached a sensitivity of the order of 10^{-16} m.

A common disadvantage of all interferometric methods used until now is the tuning of the interferometer into a regime of maximum steepness. The photo-detector is illuminated by relatively intensive dc light, and it is not possible to use powerful lasers and highly sensitive photo-detectors. As a result the sensitivity is considerably limited.

The purpose of this paper is to show how one can raise the sensitivity of measuring SPD with interferometric systems using powerful lasers and highly sensitive photo-detectors.

A. Interferometric Systems for SPD Measurements

Two-beam and multi-beam interferometric systems for SPD measurements are outlined in the upper parts of Figs. 1 (a) and 1 (b). The output light beam power P as a function of the optical path difference A at maximum contrast and coherent light source are shown in the lower part of each figure. To obtain maximum steepness the optical path difference has to be $\Delta = \Delta_1 \pm n\,l$ where $n =$ 0,1,2,... and l is the optical wavelength. The SPD amplitude Δ_o can be calculated according to[3,4,14]

$$\Delta_o^T = l P_\sim \sqrt{2} / 2pP_o \;\; (2P_2 = P_o) \tag{1}$$

for a two-beam interferometer and

$$\Delta_o^M = l(1-R)P_\sim / 8pP_2\sqrt{RP_o / P_2} \tag{2}$$

for a multi-beam interferometer when $P_o >> P_2$ as shown in Fig. 2. In Eqs. (1) and (2) P_o, P_2, and $P_\sim$ are incident, reflected dc, and ac light power components, and R is the reflection coefficient of the multi-beam interferometer mirrors. The smallest amplitude $\Delta_{\min}^T$ that can be measured at a given noise level of the apparatus is[3,14]

$$\Delta_{o,\min}^T \geq (l/p)\sqrt{e\Delta f / 2a(P_o)}P_o \tag{3}$$

where e is the charge of the electron, Δf is the passband, and $a(P)$ is the photo-detector sensitivity.

As can be been from Eq. (3) the greater the product $a(P_o)P_o$, the higher the sensitivity of SPD measurements. However, the product $a(P_o)P_o$ is considerably limited in the maximum steepness measurements because of the high dc output light. This is the main reason SPD smaller than 10^{-14} m have not been measured until now with a two-beam interferometer.

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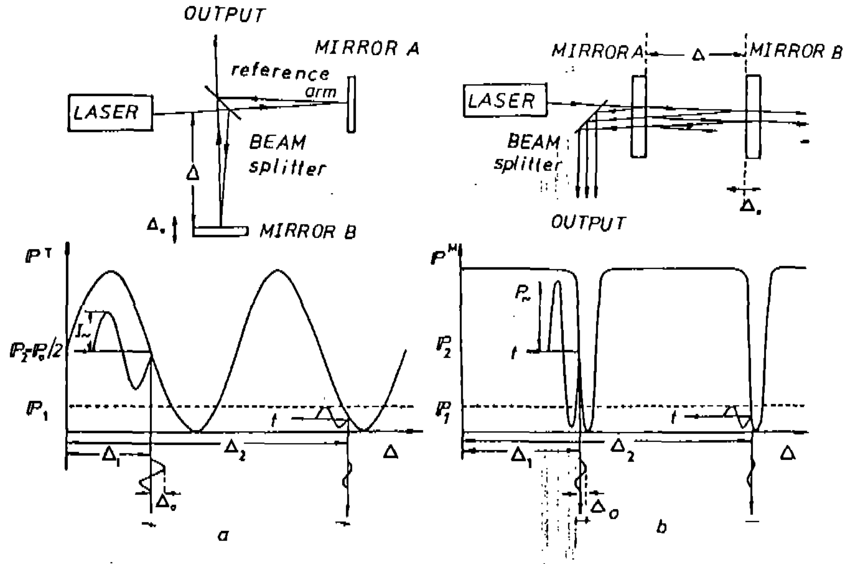

Fig. 1. (a) Above, experimental setup of the Michelson type two-arm interferometer. Below, dependence of the output light beam power P^T on the two-arm interferometer's variable arm length Δ.
(b) Above, experimental setup of a Fabry-Perot type multi-beam interferometer. Below, dependence of the output light beam power P^T on the interferometer's length Δ.

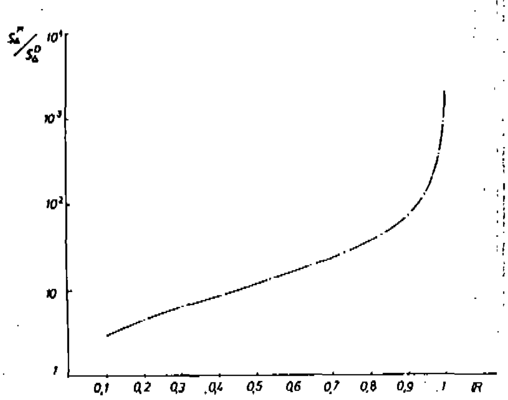

Fig. 2. Dependence of the ratio S^M_Δ / S^T_Δ between the steepness values of the two-beam and multi-beam interferometers, respectively, on the reflection coefficient R.

The same restriction is also valid for the multi-beam interferometer.

It is possible, however, with the variable arm length, to maintain an optical path difference $\Delta = \Delta_2$, such as is shown in Fig. 1, that the reflected light beam power P_2 does not became greater than that at which the photo-detector can preserve its high sensitivity $a(P_o)$ within the linear part of its lux-ampere characteristics. In this case the increase of laser power P_o will not decrease $a(P_o)$ and both the production $a(P_o)P_o$ and steepness $S_{\Delta_2} = (\partial P/\partial \Delta)_{\Delta_2}$ at point Δ_2 will increase with P_o.

B. Sensitivity of the Interferometric System

If the variation $d\Delta = \Delta_o$ in the optical path difference is sufficiently small we may retain only the first-order terms:

$$P = P(\Delta_2) + S_\Delta . \Delta_o = P_2 + P_\sim \qquad (4)$$

from the expansion in a Taylor series reflected optical beam power P. The ac component $P_\sim$ as seen from Eq. (4) depends essentially on the steepness S_{Δ_2} For multiple interference with lossless mirrors the steepness $S^M_{\Delta_2}$ is

$$S^M_{\Delta_2} = 4p\,(1 - P_2/P_o)\sqrt{P_2 P_o F - P_2^2(F+1)}\,/\,l$$
$$\cong 4p\sqrt{P_2 P_o F}\,/\,l \qquad (5)$$

because the reflected power P is related to the incident power P_o by[16]

$$P = P_o 4R\sin^2(\Phi/2)/\left[(1-R^2) + 4R\sin^2(\Phi/2)\right]. \quad (6)$$

In the above formulas F = 4R/(1 - R^2), F =2pD/l +d$_1$/2+d$_2$/2, and it must be taken into account that

$$2p\Delta_2/l + d_1 + d_2 = 2\arcsin\sqrt{P_2/F(P_o - P_2)}\,. \qquad (7)$$

Here $P_2(<<P_o)$ is the maximum power at which the photodetector preserves its linear lux-ampere characteristics, and d_1 and d_2 are the phase changes caused by the reflections from the mirrors.

It is seen from Eq. (5) that the steepness (respectively, the sensitivity) will rise infinitely with P_o without any change in the photodetector sensitivity $a(P)$. One can obtain a similar result in the case of a two-beam interferometer where the steepness S^T_Δ is

$$S^T_\Delta = p\sqrt{P_1 P_o}\,/\,l$$

(8)

The ratio between the two steepness values when $P_1 = P_2$ gives

$$S^M_\Delta / S^T_\Delta = 4\sqrt{F} = 8\sqrt{R}/(1-R) > 1 \qquad (9)$$

and it depends only on the reflection coefficient R when both interferometers work in the same conditions-equal power for registering P_2 and equal incident light beam power P_o. The ratio is shown in Fig.2 as a function of R. The advantage of multi-beam interferometry using dielectric mirrors with reflection. coefficients close to unity for measuring small displacements is obvious.

III. Noise Factors Restricting the Sensitivity

A. Shot Effect

The number of output photons per unit time is $n = P/h$. Correspondingly, the number of photoelectrons N, created by them per unit of time is given by

$$N = \eta n = \eta P_o F \sin^2(\Phi/2)/h\nu\left[1+F\sin^2(\Phi/2)\right] \quad (10)$$

where η is the quantum efficiency, h is Planck's constant, and ν is the light frequency.

A small variation Δ_o^M of the distance Δ between the mirrors provokes a phase change $d\Phi$ and therefore causes a change in the number of photoelectrons dN per unit of time:

$$dN = \eta dn = \eta P_o F \sin\Phi \pi \Delta_o^T / hc\left[1+\sin^2(\Phi/2)\right] \quad (11)$$

where c is the light velocity in vacuum.

The averaged quadratic fluctuation $\overline{(\Delta N)^2}$, the number of electrons created per unit of time in the frequency band $2\Delta f$, is given by the well-known Poisson formula

$$\overline{(\Delta N)^2} = 2N\Delta f \quad (12)$$

Obviously, for detecting the displacement it is necessary to have $dN^2 \geq \overline{(\Delta N)^2}$, i.e.,

$$(\Delta_o^M)^2 \geq \frac{l^2 h\nu\Delta f\left[1+F\sin^2(\Phi/2)\right]^3 \sin^2(\Phi/2)}{\pi^2 \eta F P_o \sin^2\Phi} \quad (13)$$

If $N_1 < P_o / h\nu$ is the maximum admissible intensity at which the photodetector preserves the linearity of its lux-ampere characteristics, the adequate phase angle at which the interferometer must be tuned is

$$\Phi_1 = 2\arcsin\sqrt{N_1 / F(P_o/h\nu - N_1)} \quad (14)$$

So that the smallest periodic displacement $\Delta_{o,\min}^M$ that can be measured in a given frequency band according to Eqs. (13) and (14) is

$$\Delta_{o,\min}^M / \Delta f^{1/2} = \frac{l\sqrt{e/aFP_o}(1-N_1 h\nu/P_o}{2\pi(1-N_1 h\nu/P_o)} \cong \frac{el}{2\pi FP} \quad (15)$$

where $a = \eta e / h\nu$ is the sensitivity in *A/W* of the photo-transducing device and $P = aP_o$.

As one can see from Eq. (15) the smaller the wavelength and the higher the sensitivity, the laser power and the mirror's reflection, the smaller the vibration detected in a unit frequency band.

If $G = a/S$ and $S = leQ(l)/hn$ are the gain and the cathode sensitivity of the photo-multiplier, Eq. (15) can be rewritten in the form:

$$\Delta_{o,\min}^M / (\Delta F)^{1/2} = (1-R\sqrt{hn l / RP_o GQ} / 4\pi \quad (16)$$

where Q is the quantum efficiency. If, for example, R = 0.975, $Q(l)$ = 10%, l = 0.6328 X 10^{-6} m, P_o = 0.1 W, and G = 10^5 the amplitude of the smallest displacement limited by shot noise is

$$\Delta_{o,\min}^M / (\Delta f)^{1/2} = 2{,}3\times10^{-20} \text{ m s}^{1/2}.$$

However, the increase of the laser power and the photo-multiplier gain G require high contrast because the power that can be scattered on the photo-multiplier's anode and the maximum value of the anode's current $J_{\max}$ in the linear part of the lux-ampere characteristics are limited. The maximum admissible incident power of the light beam $P_2^{\max}$ is related to $J_{\max}$ and the other parameters of the photo-multiplier through the formula

$$GP_2^{\max} = J_{\max} hc / lQ(l)e. \quad (17)$$

If $C = P_{\max} / P_{\min} = \left[(1+R)/(1-R)\right]^2$ is the interferometer's contrast, where $P_{\max}$ and $P_{\min}$ are the maximum and the minimum values of the reflected light beam power, from Eq. (17) one can obtain

$$Q(l)P_o G = \left[(1+R)/(1-R)\right]^2 J_{\max} hc / xle \quad (18)$$

where $x = P_2^{\max} / P_{\min} \geq 1$. In Eq. (18) we assume that $P_{\max} \cong P_o$ and it has taken into account that $P_2^{\max} = xP_o / C$. As one can see when utilizing the high power values P of the light beam, the high amplification G and the high quantum efficiency $Q(l)$ depends only on the interferometer's contrast C. It can increase to C^p if a complementary device assuring p times trip of the light beam through the interferometer is used, and from Eqs. (17) and (18) one can obtain for $\Delta_{o,\min}^M / (\Delta f)^{1/2}$,

$$\Delta_{o,\min}^M / (\Delta f)^{1/2} = l(1+R)\sqrt{xe / RJ_{\max}} / 4\pi C^p \quad (19)$$

If a device assuring one or three trips of the light beam through the interferometer is used and if we assume that x =2, l = 0,488x10^{-6} m $J_{\max}$ = 10^{-3} A, R= 0.975, we obtain the amplitude of the smallest vibration which can be detected consecutively:

one trip: $\Delta_{o,\min}^M / (\Delta f)^{1/2} = 2.23\text{x}10^{-19}$ m $s^{1/2}$,

three trips : $\Delta_{o,\min}^M / (\Delta f)^{1/2} = 5.7\text{x}10^{-27}$ m $s^{1/2}$.

The above-mentioned values according to Eq. (17) can be reached if lasers and photo-multipliers having the following parameters are used:

One trip: $Q(l)$ =30% P_o = 1W G = 26,

Three trips: $Q(l)$ =30% P_o = 8 W G = 10^8.

As one jean see from this analysis the contemporary possibilities of optical, laser, and phototransducing techniques allow one to reduce considerably the influence of the shot noise over the measurement of very small displacements up to 5.7 x 10^{-27} m $s^{1/2}$.

In Fig. 3 the ratio $\Delta_{o,\min} / (\Delta f)^{1/2}$ as a function of $P = aP_o$ in the case of a two-beam and a multi beam interference with R = 0.99 is shown. The smallest vibrations measured optically until now are noted by circles, triangles, and asterisks. As one can see there are unex-ploited possibilities to measure small displacements by using high values of P. Unfortunately, as we shall see, the increase of power P_o of the laser beam is accompanied by a competing process which cannot be overcome and which provokes the decrease of the interferometric system sensitivity. This process is related

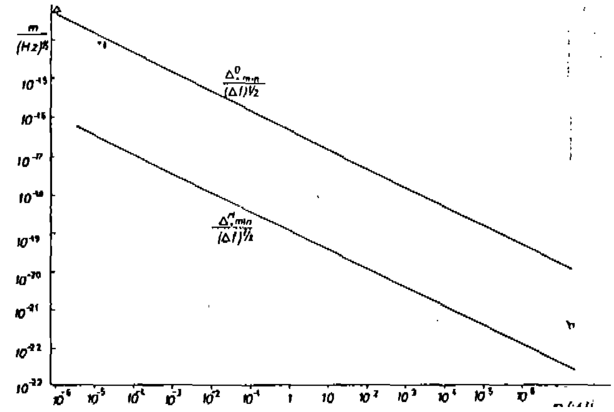

Fig. 3. Ratio $\Delta_{o,\min}/(\Delta f)^{1/2}$ *as a function of* $P = aP_o$ *and multi-beam interferences with* R *= 0.99.*

to the increase in the fluctuations of radiation pressure acting on the optical system's mirrors when the laser beam power increases.

B Radiation Pressure Noise

The force T **of the radiation pressure** P **exerted by a laser beam with a cross section** S **on a surface with reflection coefficient** R **is**

$$T = SP = nhn(1+R)/c = P_o(1+R)/c \qquad (20)$$

The mean quadratic fluctuation $\overline{(dn)^2}$ **of the number of photons emitted by the laser per second in the** $2\Delta f$ **frequency band is given by a formula similar to Eq. (12):**

$$\overline{(dn)^2} = 2n\Delta f \qquad (21)$$

Therefore an alternative component of the force will be exerted on the mirror's surface, besides the continuous component of this force [Eq. (20)] in the frequency band whose mean quadratic amplitude value $\overline{(dT_o)}^2$ **is**

$$\overline{(dT_o)}^2 = 2n\Delta f h^2 n^2 (1+R)^2/c^2 = 2T^2 hn\Delta f/P_o \qquad (22)$$

The ac component will provoke a variable mechanical deformation of the mirror surface with the same frequency w **and with an amplitude** U_o **that, according to Hook's law, is**

$$\Delta T_o/S = EdU_o/dx = EkU_o\exp(ip/2) \qquad (23)$$

Here E **is Young's modulus and** $k = w/V_L$ **the wave-number of the longitudinal waves propagating with velocity** $V_L = \sqrt{E/r}$ **. The phase term in Eq. (23) shows that the deformation and the force creating this deformation are phase shifted at** $p/2$**. The mean quadratic value of the mirror surface deformation in a unit frequency band is given by**

$$\frac{\overline{(\Delta U)^2}}{\Delta f} = \left(\frac{V_L}{SEw}\right)^2\frac{hn}{P_o} = \left[(1+R)^2/Scw\right]\frac{2hnP_o}{rE} \qquad (24)$$

As can be seen from Eq. (24) the increase of the laser beam power P_0 **causes an increase of** $\overline{(\Delta U)}^2/\Delta f$ **in contrast to** $\Delta_{o,\min}/(\Delta f)^{1/2}$ **that decreases, according to Eq. (15).**

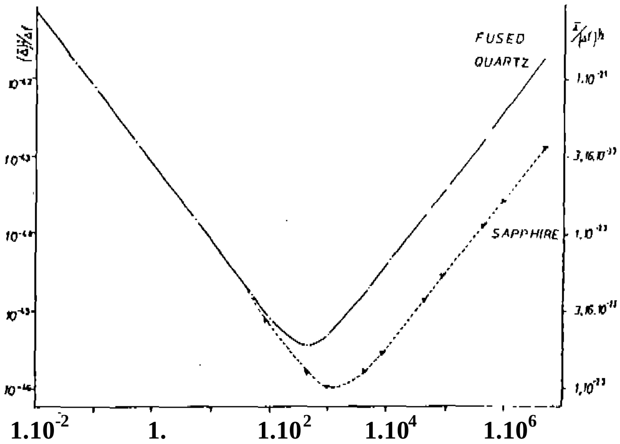

Fig. 4. Dependence of the total noise mean square in a unit frequency band $\overline{(\Delta)^2}/\Delta f$ *on the laser beam power* P_o*.*

The square of the total noise $\overline{(\Delta)}^2/\Delta f$ **in a unit frequency band limiting the smallest vibration amplitude which can be measured in the present conditions will be the sum of two competing processes described by Eqs. (15) and (24), i.e.,**

$$\overline{(\Delta)}^2/\Delta f = (\Delta_{o,\min})^2/\Delta f + \overline{(\Delta U)}^2/\Delta f =$$
$$(1+\frac{R}{Sw})^2\frac{2hP_o}{r\,l\,c} + l\,hc(1-R^2)/16pRP_o \qquad (25)$$

In Fig. 4 $\overline{(\Delta)^2}/\Delta f$ **as a function of the laser beam power** P_o **is shown for a fused quartz mirror with** r **= 2.2 x 10**^{3} **kg/m**^{3}**,** E **= 7.8 X 10**^{10} **N/m**^{2}**,** R **= 0.99,** $f = w/2p$ **= 10**^{3} **Hz, and** h **= 10**^{6}**. As can be seen from Fig. 4 the experiment can be optimized by using _values of the laser beam power** $P_o = P_o^{\min}$ **at which** $\overline{(\Delta)}^2/\Delta f$ **has a minimum value of** $\overline{(\Delta)}^2_{\min}/\Delta f$ **. These values for fused quartz mirrors and for a 0.01% frequency band** $\Delta f/f$ **are**

$$\frac{P_o^{\min}}{S} = l\,cw\sqrt{\frac{rE}{2hF}}/2p(1+R) = 444\text{x}10^4 \text{ W/m}^2, \qquad (26)$$

$$\overline{(\Delta)}^2 = h(1-R^2)\frac{\Delta f}{f}2p^2S\sqrt{2hRrE} =$$

=(8.5x10^{-23} **m)**^{2} **(27)**

C. Heat Fluctuation of the Mirror Surface

The atoms ceaselessly vibrate around the equilibrium positions (with frequency in a wide band from the lowest to the Debye frequency and with amplitudes depending on temperature. Even at absolute zero temperature the atoms vibrate because of the quantum nature of the linkage among them. The low frequency vibrations participate as noise and therefore limit measurement of very small displacements. In reality these are plane longitudinal waves with the wave vector perpendicular to the mirror surfaces. To evaluate the amplitude of these noise vibrations it is important to establish its dependence on the temperature and on the mirror material parameters.

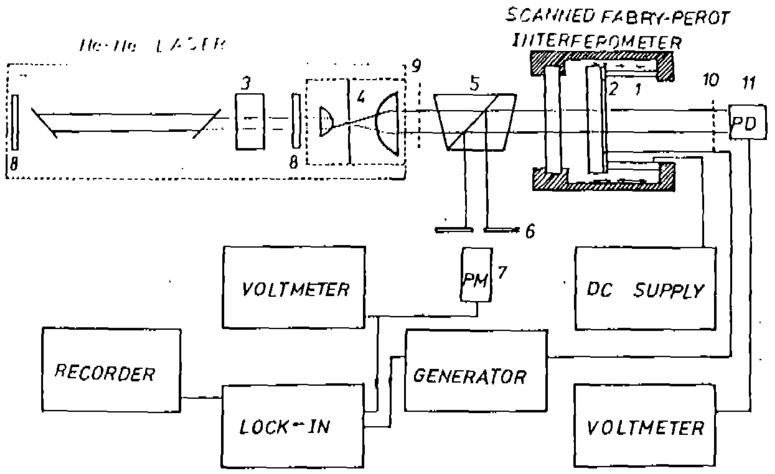

Fig. 5. *Experimental setup for measuring very small periodic displacements with a multi-beam interferometric phase bridge.*

The heat part of the internal energy density of a solid at a temperature T is given by

$$u - E_o = \frac{N}{V}\int_o^{\infty} g(f)hf\exp(-\frac{hf}{kT})\left[1-\exp(-\frac{hf}{kT})\right]^{-1} df, \quad (28)$$

where u and E_o are the density of the total energy of the solid and the energy of the vibrations at absolute zero temperature, respectively, k is the Boltzmann constant, f is the frequency, N is the number of atoms in volume V, and $g(f)$ is the frequency distribution function.

For the long-wave range of the vibrational spectrum Debye's theory is the most appropriate in the determination of $g(f)$. According to this theory the heat energy is carried by standing longitudinal and transverse elastic waves propagating with velocities V_L and V_T, respectively, and the frequency distribution function is

$$g(f) = 4p\frac{V}{N}\left[1/V_L^3 + 2/V_T^3\right]f^2 \quad (29)$$

The crystal's internal energy density carried only by longitudinal waves in the frequency band $f \pm \Delta f$ according to Eqs. (28) and (29) is given by

$$u - E_o = \frac{4pkT}{V_L^3}\int_{f-\Delta f}^{f+\Delta f}\left[1 - \frac{x}{2} + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}B_n x^{2n}}{(2n)!}\right]f^2 df \quad (30)$$

where $x = hf/kT$ and B_n are Bernoulli's numbers. Under the integral, the function $x/(e^x - 1)$ is developed in a Taylor series. For frequencies up to 10^6 Hz and temperature values higher than 10^{-3} K, x « 1, and in the series one can keep only the first-order terms. In this way, for the energy density $(u - E_o)_f$ at a frequency f in the frequency band $2\Delta f$ one obtains

$$(u - E_o)_f = 2\frac{kTp}{V_L^3} f^2(1 - \frac{hf}{kT})\Delta f \cong \frac{2pkTf^2\Delta f}{V_L^3} \quad (31)$$

However, this energy is related to the density r, to the frequency f, and to the amplitude $d_{o,f}$ of the longitudinal wave according to

$$(u - E_o)_f = \frac{1}{2\Delta f}\int_{f-\Delta f}^{f+\Delta f} W_f df = 6p^2 r f^2 d_{o,f}^2 \quad (32)$$

where $W_f = 2p^2 r f^2 r_{o,f}^2$ is the longitudinal wave energy density. From the above formulas for the heat noise vibration amplitude Δ_N^T limiting the smallest measurable vibration amplitude $d_{o,f}$ on can obtain

$$\Delta_N^T = d_{o,f} = \sqrt{kT\Delta f / prV_L^3} \quad (33)$$

The most appropriate materials for Fabry-Perot interferometer mirrors are those with high density and high propagation velocities of the elastic waves as, for example, sapphire. The noise vibration amplitude at room temperature and in the 0.01-Hz frequency band for sapphire and quartz mirrors according to Eq. (34) are 4.3 x 10^{-20} and 1.72 x 10^{-19} m, respectively.

IV. Experimental Setup and Results

Figure 5 shows the experimental setup for measuring small periodic displacements with a multi-beam interferometric phase optic bridge. The multi-beam interferometer used was a scanning Fabry-Perot with mirrors of fused quartz and dielectric layers for wavelength l = 0.6328 x 10^{-6} m. The reflection coefficient of the mirrors is R = 97.5% and its flatness is $F_F = l/250$. The solid metal construction supporting the mirrors has a thermal expansion coefficient g = 0.36 x 10^{-6} deg^{-1}. Piezoelectric disks (1), mechanically independent and electrically connected in parallel, are used for a fine displacement with an accuracy of 1.25 x 10^{-9} m/V. Three other x-cut plates of a-quartz (2) with metal electrodes connected in parallel are used to vibrate the movable mirror at amplitude Δ_o and frequency Ω. A He-Ne laser with a power of 30 mW is used. To improve the monochromaticity and the laser beam structure a constant Fabry-Perot interferometer (3) tuned to wavelength l = 0.6328 x 10^{-6} m was built into the laser as well as a collimator tube (4) with a 6 m*m* hole for space filtration.

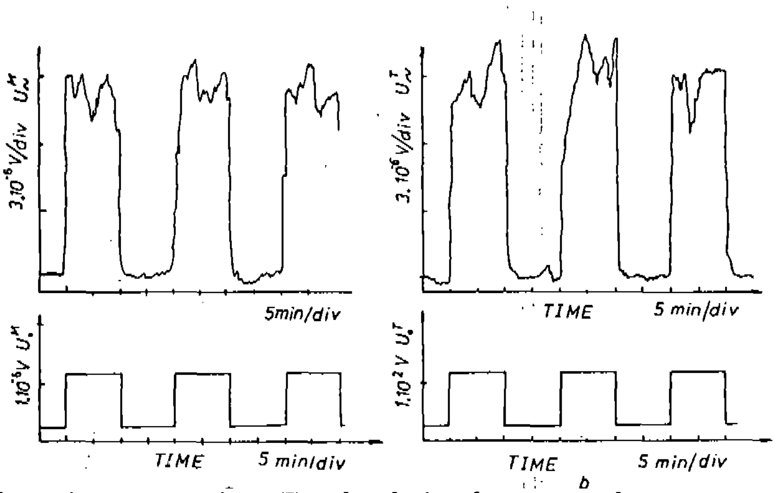

Fig. 6. (a) Multi-beam interferometer and (b) two-beam-interferometer. Above (a) and (b), time dependences of the voltages $U_\sim^M$ and $U_\sim^T$ (I UL in the synchronous voltmeter's output. Below (a) and (b), time dependences of the variable voltages U_o^M and U_o^T applied to quartz .plates.

The semitransparent mirror (5) makes the interferometer work in a reflected light regime. The diaphragm (6) selects the most suitable zone of the light field. The photo-multiplier (7) with a high amplification coefficient G transforms the light beam passing through the diaphragm (6) into an electric signal. A low frequency synchronous-selective technique is used to detect the electric signals. The grey optic filters (9) and (10) are used to reduce the laser beam power and to tune the interferometer in a regime of maximum steepness. The chosen work mode of the interferometer at point Δ_1 or Δ_2 is controlled by the photodiode (11) and the small fluctuations of its arms are neutralized by the piezoelectric disks (1).

The multi-beam interferometric optic bridge with one movable mirror can be easily transformed into a one-arm optic bridge using Newton fringes[14,15] by replacing the fixed mirror with a flat concave lens. This allows us to compare the two methods in the same conditions—to measure small periodic displacements at equal laser beam powers P_o and at maximum steepness of the interferometers.

A. Measurement in a Regime of Maximum Steepness

When $\partial^2 P/\partial\Phi^2$ =0 and Eq. (6) is used for the phase angle Φ_{max} corresponding to the maximum sfeepness $S_{\Delta_2} = (\partial P/\partial\Phi)(\partial\Phi/\partial\Delta)$ of the multi-beam interferometer, one can obtain

$$\cos\Phi_{max} = -b + (b^2+2)^{1/2}, \qquad (34)$$

where $b = (2+F)/2F$. The corresponding output power P_2^{max} is.

$$P_2^{max}/P_o = (1-\cos\Phi_{max})(2/F+1-\cos\Phi_{max}) = \frac{3F+2-\sqrt{9F^2-4F+4}}{3F+6-\sqrt{3F^2+F+4}} \qquad (35)$$

As can be seen from Eq. (35), at a given input power P_o of the laser beam, P_2^{max} depends on the quality of the reflection F (respectively, on the reflection coefficient R) of the interferometer. For example, if R = 0.975 the ratio P_2^{max}/P_o is 0.2499. This is used to tune the multi-beam interferometer to a regime of maximum steepness: at an angle $\Phi = p$ by using a gray filter the input power P_o of the laser beam is reduced to the maximum admissible power at which the photo-multiplier still preserves its linear lux-ampere characteristics. Then the distance Δ between the mirrors is slowly changed by the piezoelectric disks (2) until one obtains a new value J_2^{max} of the anode current of the .photo-multiplier satisfying the ratio $J_2^{max}/J_o = P_2^{max}/P_o$ = 0.2489. When one of the mirrors vibrates with small amplitude Δ_o^M it is determined according to the expression

$$\Delta_o^M = l(1-R)P_\sim/8pP_2^{max}\sqrt{RP_o/P_2^{max}} \qquad (36)$$

obtained from Eqs. (2) and (35). From Eq. (36) one can see that measuring the small vibration amplitude Δ_o^M depends on the possibility of measuring the small ratios $P_\sim/P_2^{max}$ between the ac light power $P_\sim$ due to the small vibration and the output light power P_2^{max} of the interferometer.

In Figs. 6(a) and (b) the upper curves represent the registered voltages $U_\sim^M$ and $U_\sim^T$ in the output of the synchronous voltmeter in the frequency Δf = 0.01 Hz for multi-beam and two-beam interferometers with equal input laser beam power. The lower curves represent the ac voltages U_o^M and U_o^T applied to the quartz plates (2) to make the mirrors vibrate at the same ratio $P_\sim/P_2^{max}$ in both cases. To obtain $U_\sim^M \cong U_\sim^T$ as seen in Fig. 6 it is necessary to apply voltages on the quartz plates (2) with amplitudes giving a ratio of $U_o^M/U_o^T \cong 10^{-4}$. The value of this ratio is explained by the fact that the same ratio value must hold between the vibration amplitudes Δ_o^M and Δ_o^T of the mirrors, i.e., $U_o^M/U_o^T = \Delta_o^M/\Delta_o^T$, because the piezoelectric effect is linear. Having in mind Eqs. (1) and (36) as well as R = 0.975 for the ratio Δ_o^M/Δ_o^T, one actually obtains

$$\Delta_o^M/\Delta_o^T = (1-R)/2\sqrt{2RP_o/P_o^{max}} \cong 10^{-4} \qquad (37)$$

The smallest vibration amplitude that was measured by the one-arm optic bridge as of the order of 3 x 10^{-14} m. The same ratio $P_\sim/P_2^{max}$ was also achieved with the multi-beam

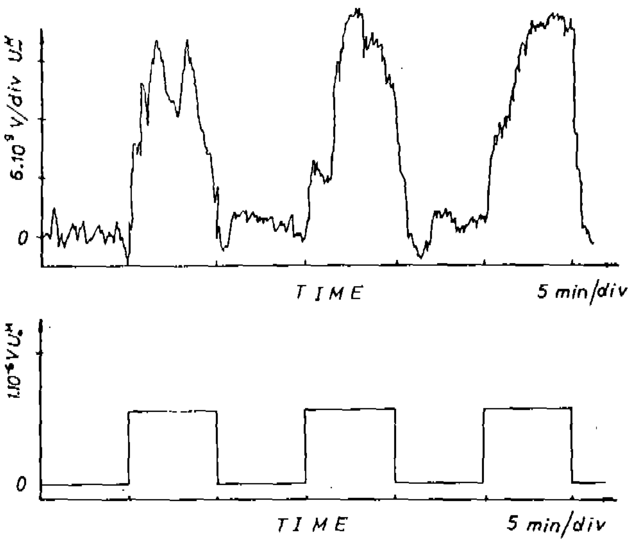

Fig. 7. Above, time dependence of the registered voltage $U_{\sim}^{M}$ by the synchronous voltmeter in the photo-multiplier's output in an f = 0.01-Hz frequency band. Below, time dependence of the variable voltage supplying the quartz plates to make the mirror vibrate.

interferometric bridge, so that a vibration amplitude 10^{-4} times smaller, i.e., of the order of Δ_{o}^{M} = 3 x 10^{-18} m was measured.

B. Measurement in a Regime of Minimal Constant Lightening

An input laser beam power P_o of the order 16 x 10^{-3} W is used for measuring in a regime of minimum constant lightening. By tunning the interferometer one can obtain a maximum light beam power value $P_{\max}$ on the photo-multiplier of the order of 7.8 x 10^{-3} W as well as a minimum value $P_{\min}$ of the order of 1.6x10^{-6} W. A photo-multiplier admitting a maximum anode current of the order of 10^{-3} A and with a gain of the order of 10^3 - 10^4 is used. In the anode circuit a loading resistance R = 10^3 W is switched on. The maximum admissible power $P_2^{\max}$ of the light beam incident on the photo-multiplier cathode up to which the device preserves its high amplification is of the order of 2.6x10^{-5} W. This value is obtained by tuning the interferometer at point Δ_2 as shown in Fig. 1. In Fig. 7 the upper curve represents the voltage $U_{\sim}^{M}$ recorded by the synchronous voltmeter in the output of the photo-multiplier in a frequency band of Δf = 0.01 Hz and the lower curve represents the ac voltage supplying the quartz plates (2) for making the mirror vibrate.

If we assume that the recorded value of the ac voltage is $U_{\sim}^{M}$ = 17x10^{-9} V from Eq. (2) one can calculate the vibration amplitude value Δ_{o}^{M} = 7.2x10^{-19} m. This amplitude is greater than the noise vibration amplitude calculated with Eqs. (19), (24), and (33). Moreover since $U_{\sim}^{M}/U_{2}^{\max} = P_{\sim}/P_{2}^{\max}$ = 2x10^{-8} the light beam amplitude that we have measured would be equal to $P_{\sim}$ = 5.2x10^{-13} W. This value is, by several orders, greater than the equivalent light noise component of the used photo-cathode.

V. Conclusion

The experimental results and the analysis show that multi-beam interferometer, due to its high steepness, allows one to measure very small periodic displacements of the order of 10^{-19} m even at room temperature. Among the noise restricting the sensitivity at room temperature the most important is heat fluctuation of the mirror surface. The heat fluctuations depend on the material constants (density and elastic moduli) of the Fabry-Perot mirrors. At low temperatures this noise component decreases, and shot noise and radiation pressure noise dominant. The amplitude of this total noise is minimal at a given laser beam power.

The regime of a minimal constant lightening $(\Delta = \Delta_2)$ allows one to raise the measurement sensitivity. with a Fabry-Perot interferometer using powerful lasers and highly sensitive detectors.

Contemporary optics, photo-electronics, and laser, techniques in general make it possible to build an optic measurement system to determine very small periodic displacements up to 10^{-23} m. This has a considerable potential for uses in cosmology and relativity theory.

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