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One-Arm Phase Optical Bridge for Measuring Small Amplitude High Frequency Vibrations

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In this paper a one-arm phase optical bridge with axial symmetry which utilizes Newton's interference fringes is described and its application for measuring small amplitude high frequency vibrations (1-100 MHz) is also given. With a preliminary modulation of the laser beam and making use of the heterodyne method, a simple low frequency lock-in amplifier can be used for lowering the noise. A high sensitivity of about 10^{-11} m is obtained. An additional low frequency modulation of the length of the arm of the bridge, makes it possible to avoid the very difficult problem of maintaining the constant length of the arm, without lowering the sensitivity and the precision of the measurements. In this way the scanning, measuring and registration of the distribution of the vibrations having a component in the direction of the axis of the phase bridge are proved to be very simple.

The one-arm phase optical bridge with axial symmetry (utilizing Newton's rings) for measuring small mechanical vibrations has many advantages over the two-arm bridges.[1)] With such a bridge small periodic vibrations in the sound band were measured.[1)] A considerable widening of the frequency region is obtained when the laser beam intensity I is modulated before falling onto the bridge according to the relation

$$I = I_0(1+b\sin w_1 t) \tag{1}$$

where $0 \le b \le 1$ is the modulation coefficient.

The block diagram of the experimental set-up for measuring of small high frequency vibrations is shown in Fig. 1. The surface of the AT-cut quartz plate, whose vibrations* are to be measured and the surface of the lens L with a large radius of curvature R are separated by an air gap of thickness *d* through which the coherent light of a He-Ne laser passes. To obtain an equal intensity of the interfering beams the convex surface of L is covered with a thin aluminium film with reflectivity of about 38 %.

*The vibrations measured are those of flexure with amplitude normal to the plane of the plate which corresponds to the principal thickness-shear mode displacement.[2)]

If the plate 1 vibrates with angular frequency w_2

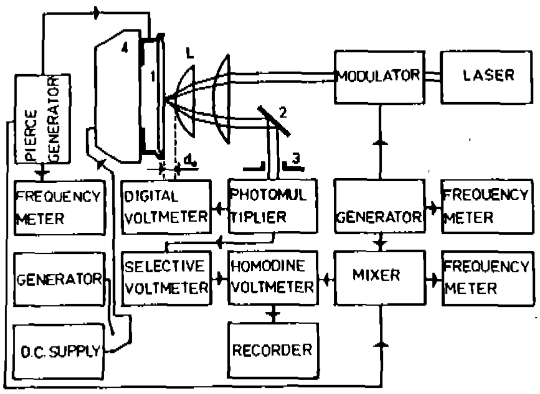

Fig. 1. Block-diagram of the experimental equipment: 1-quartz plate, 2-mirror, 3-mask, 4-telephone membrane.

and d changes according to the law

$$d = d_0 + d_0' \sin w_2 t \tag{2}$$

the light flux Φ, which passes through the blind 3 with a hole of radius r_o and falls onto the cathode of a photoelectric multiplier, shows maximum modulation[1)], when $d_o = (n+1)(l/4)$, n = 1, 2, ... The light flux Φ is given by

$$f = \frac{p}{2} I_o R l (1 + b\sin w_1 t) + I_o R l (1 + b\sin w_1 t)\sin\left[\frac{4p}{l} d_o + d_o' \sin w_2 t\right] \tag{3}$$

where r_o^2 is replaced by $Rl/2$. If in (3) we separate the terms describing the d. c. and a. c. components of the light flux Φ we obtain:

$$f = \frac{p}{2} I_o R l + \frac{p}{2} I_o R l b \sin w_1 t + (-1)^{n+1} I_o R l \sin(m \sin w_2 t) + (-1)^{n+1} I_o R l b \sin w_1 t \sin(m \sin w_2 t), \tag{3'}$$

where $m = (4p / l)d_o$. Expanding $\sin(m \sin w_2 t)$ in a Fourier series(3') becomes:

$$f = \frac{p}{2} I_o R l + \frac{p}{2} I_o R l b \sin w_1 t + (-1)^{n+1} I_o R l \left\{ 2 \sum_{k=1}^{\infty} J_{2k-1}(m) \sin[(2k-1)w_2 t] \right\} + \\ (-1)^{n+1} I_o R l b \left\{ \sum_{k=1}^{\infty} J_{2k-1}(m)[\cos[w_1 - (2k-1)w_2 t]t - \cos[w_1 + (2k-1)w_2]t] \right\} \quad (4)$$

were $J_r = (\frac{m}{2})^r \sum_{q=0}^{\infty} \frac{(-1)^q}{q!\Gamma(q+r+1)} \left(\frac{m}{2} \right)^{2q}$, are the Bessel functions of order (r= 1, 2, 3, . .).

It is seen that apart from the constant component $(p/2) I_o R l$ the light flux falling on the catode of the photoelectric multiplier contains also variable components with frequences w_1, w_2, $w_1 - w_2$, $w_1 + w_2$ and so on.

The output voltage u which is obtained accross the load resistance R_T of the PMT is given by an equation similar to (4):

$$u = u_= b \sin w_1 t + ... \\ + u_\sim b J_1(m) \cos(w_1 - w_2) t + ...,$$

where $u_=$ and $u_\sim$ are given by

$$u_= = a I_o R l \frac{p}{2}, \quad u_\sim = a I_o R l,$$

(a is the efficiency coefficient of the PMT). The output voltage u involves a. c. components with frequencies $w_1, w_2, w_1 - w_2, w_1 + w_2$, etc.

If the effective value $u_{eff}(w_1 - w_2) = u_\sim b J_1(m)$ of the component of u with frequency $w_1 - w_2$ is measured by means of a lock-in amplifier for small vibrations ($m << 1$, $J_1(m) = m/2$) the amplitude of vibration of the quartz plate can be calculated by the equation:

$$d_o' = \frac{l}{pb\sqrt{2}} \frac{u_{eff}(w_1 - w_2)}{u_{max} - u_{min}}$$

where $u_\sim = (u_{max} - u_{min})/2$ and u_{min} are given by (8) in the following. When the quartz plate is not vibrating and the light flux is not modulated $w_1 = w_2 = 0$, from (3) and (5) it is obtained

$$u = u_= + u_\sim \sin \frac{4p}{l} d_o \quad (7)$$

Placing the plate 1 to $d_{o1} = (n + 1/4) l / 2$ and $d_{o2} = (n + 3/4) l / 2$, the voltage across the load resistance R_T of the photoelectric multiplier takes the values respectively

$$n(d_{01}) = u_= + u_\sim = u_{max},$$

$$u(d_{o2}) = u_= - u_\sim = u_{min}, \quad (8)$$

which correspond to the conditions of maximum and minimum modulations. From (8),

$$u_=(r_o, d_o) = \frac{u_{max} + u_{min}}{2} \quad (9)$$

is also obtained. The smallest amplitude d'_{min} that can be measured at the given noise level of the apparatus, is

$$d'_{o\,min} \geq \frac{l \sqrt{e u_=(r_o, d_o) \Delta f R_T}}{pb(u_{max} - u_{min})} \quad (10)$$

which is obtained from (6) equating the mean value of the amplitude $u_o(w_1 - w_2)$, $\overline{u_o^2}(w_1 - w_2) =$

$$\left[\sqrt{2} u_{eff}(w_1 - w_2)\right]^2 / 2 = u_{eff}^2(w_1 - w_2) =$$

$\left[d'_{o\,min} pb(u_{max} - u_{min}) / l\right]^2$ to the mean value of noises of the PMT and the electronic equipment $u^2 = 2 e u_=(r_o d_o) \Delta f R_T$. In (10) e is the charge of the electron and Δf the pass band. It is seen that the smallest amplitude to be measured in this case is inversely proportional to the modulation coefficient b. The examined quartz plate 1 with vacuum deposited metal electrodes on both surfaces is coupled to a generator (with frequency of vibration) w_2=1311 kHz. By means of elastic, larger metal clamps to which the alternating voltage is applied, and the plate is fixed tight to a telephone membrane are isolated to each other. The receiver itself is set firmly on a coordinate table with micrometer screws allowing translation in three orthogonal directions to a precision of 5 µm. The gap d_o is regulated roughly with one of the screws. The precise regulation of d_o to the value which satisfies the conditions of maximum modulation is done by varying the current through the coil of the telephone receiver. The second quartz plate is coupled to a generator, whose alternating voltage of frequency w_1 = 1333 kHz is applied to electro-optical modulator. The modulation coefficient can be regulated by means of regulation of the output voltage of the generator. The signals of the two quartz generators are fed to a mixer from where after filtration a signal of frequency $w_1 - w_2$ is obtained which goes to a homodyne voltmeter. A photoelectric multiplier is used as photo-detector with both direct and alternating current outputs for parallel measurement of the direct and the alternating voltage across R_T of the multiplier.

The effective value of the voltage u of frequency $w_1 - w_2$, is measured with a selective voltmeter. For measurements of very small vibrations (under 0.1 A) a homodyne voltmeter is also used to set a lower noise level. During the measurement of $u_{eff}(w_1 - w_2)$ the distance d_o between the plate and the convex surface of the lens must be kept equal to an integer multiple of λ/4, i.e. the d.c. component of the voltage from the PMT (measured by means of a digital voltmeter with an accuracy of 5.10^{-3} %) must not deviate from the value given by equation (9). The small slow change in the d.c. voltage component caused by some random fluctuation of d_o will change the d.c. current through the coils of the telephone membrane. This will cause a small deformation of the membrane, which will compensate the random fluctuation of d_o and consequently the change in the d.c. voltage component of the PMT vanish. Curve 1 on Fig. 2 shows the distribution of the amplitudes of vibration of the quartz plate along a

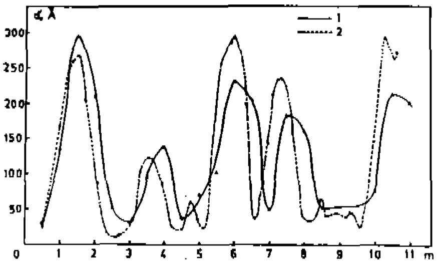

Fig. 2. Distribution of the amplitude of vibrations along a line on the surface of the quartz plate: 1-measurements were made in points of the line at 0.5 mm intervals, 2-scanning along the same line.

horizontal line. The measurement is done in points of this line 0.5 mm apart from each other.

The main difficulty in measuring the distribution of the amplitudes of vibration by the described method is in satisfying the maximum modulation condition, i.e. in keeping d_o constant to an accuracy of 0.01 µm during the scanning. If the micrometer screw translating the plate in a horizontal line is connected to a recorder a continuous measurement of d_o can be accomplished along a given line on the surface of the specimen.

The above mentioned difficulty can be avoided if a sine voltage of low frequency w_3 = 0.1 Hz ($w_3 \ll w_1 - w_2$) **is** applied to the coils of the telephone receiver. Then the gap between the lens and the plate will be governed by eq.(11):

$$d = d_o + d_o^{'} \sin w_2 t + D_o \sin w_3 t \,, \qquad (11)$$

where D_o is the amplitude of vibration of the membrane. In this case the voltage across the resistance R_t of the photoelectric multiplier from (3), (4) and (5) will be

$$u = u_{=}(1 + b \sin w_1 t) + u_{\sim}(1 + b \sin w_1 t)$$

$$\mathrm{x} \sin\left[\frac{4p}{l}(d_o + d_o^{'} \sin w_2 t + D_o \sin w_3 t)\right] \quad (12)$$

The voltage of frequency $w_1 - w_2$ can be separated with a selective voltmeter. After simple transformations it can be obtained from (12)

$$u(w_1 - w_2) = u_{\sim} b J_1(m) \cos\left[\frac{4p}{l}(d_o + D \sin w_3 t)\right]$$

$$\mathrm{x} \cos(w_1 - w_2)t. \qquad (13)$$

It is seen that the amplitude of the voltage with frequency $w_1 - w_2$ is modulated at a low-frequency w_3 and the amplitude is equal to that of eq. (5) when

$$\cos\left[\frac{4p}{l}(d_o + D_o \sin w_3 t\right] = 1 \text{, (14) , which is possible if } t = \frac{1}{w_3}\arcsin\left[\frac{1}{D_o}(\frac{li}{2} - d_o)\right] \text{ (14')}$$

The function (14) is a periodical function of t with period $T = 2p / w_3$ for the latter taking the value of unity n-times. The number i = 1, 2, 3 , . . . , *n* determines in every moment the order of the additional interference maximums (min.) due to the periodical displacement of the membrane of the telephone membrane. Its maximal value $n = 2D_o / l$ shows how many times the amplitude of vibration of the membrane is greater then the half-wavelength.

Expanding[3)]

$$\sin\left[\frac{4p}{l}D_o \sin w_3 t\right] = \sin(z \sin w_3 t)$$

$$\cos\left[\frac{4p}{l}D_o \sin w_3 t\right] = \cos(z \sin w_3 t)$$

in a Fourier series, (13) becomes

$$u(w_1 - w_2) = bu_{\sim} J_1(m) \cos(w_1 - w_2)t \left\{\cos\frac{4p}{l} d_o \left[J_o(z) + 2\sum_{k=1}^{\infty} J_{2k}(z) \cos 2kw_3 t\right]\right\} \text{ (15)}$$

where J_r is given by (4). Equation (15) shows that the spectrum of the signal $u(w_1 - w_2)$ contains besides w_3, higher harmonics components with frequencies $2w_2$, $3w_3$, $4w_3$, and so on. Consequently the amplitude and the frequency w_3 of the signal applied to the coils of the receiver must be chosen in such a way, that the spectral width of the signal (15) does not exceed the pass band of the selective voltmeter.

If together with the measurement (recordering) of u_{eff} one measures (records) by means of a d.c. voltmeter the maximum $u_{\max}$ and minimum $u_{\min}$ value of the component

$$u = u_{=} + u_{\sim} \sin\left[\frac{4p}{l}(d_o + D_0 \sin w_3 t\right] \quad (16)$$

of the voltage (12), $u_{\sim}$ is determined by the equation:

$$u_{\sim} = (u_{\max} - u_{\min})/2 \quad (17)$$

Consequently for the measurement of the amplitude of vibration of the plate it is necessary to measure (record) simultaneously the voltages (13) and (16). In Fig. 3 the recordered curves of the voltages (13) (lower curve) and (16) (upper curve) are shown as a function of the time for one point of the vibrating plate. From the lower curve, for small vibration $d_o^{'} << \lambda$ at the point in which $\cos[4p / l(d_o + D_o \sin w_3 t)] = 1$ one obtains from (13) for the amplitude of vibration

$$d_{oi}^{'} = \frac{l}{\sqrt{2}pb}\frac{u_{effi}}{u_{\sim i}} \quad (18)$$

where i—l, 2, 3, . . . , *n* determines in every moment the order of the additional maximums (min.) due to the periodical displacement of the membrane with amplitude D_o. Consequently the amplitude of vibration d_{oi} presents the t-th measurement of the amplitude of vibration of the platelet, determined by the

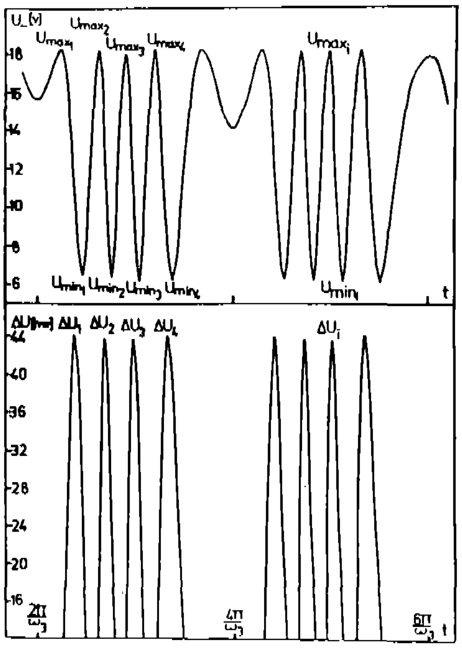

Fig. 3. Recorder registration of the voltages $u_{\sim i}$ and Δu_i used for calculation of the amplitude of vibrations $d_o^{'}$ for one point on the surface of the quartz plate .

i-th interference maximum $u_{\max i}$, i-th interference minimum $u_{\min i}$ and the effective value of the a.c. component with frequency $w_2 - w_1$, measured in the point $t_i = (1/w_3)\arcsin\left[(1/D_o)(l/2i - d_o)\right]$ for which it has maximum.

From the upper curve in the same Figure one can obtain $u_{\sim i} = (u_{\max i} - u_{\min i})/2$. The slow random fluctuations of d_o as well as those which would occur on slowly scanning along a line on the surface of sample would not change both the value of u_{effi} measured in the points in which eq. (14) is satisfied and the value of $u_{\sim i}$, calculated from eq. (17).

From the simultaneous recordings of the values of (13) and (16) by scanning the surface of the quartz plate along one line, the amplitudes of vibration along this line are calculated from (6) (curve 2 on Fig. 2). The line is chosen in such a way, that it passes through the points of the .plate surface, whose amplitudes of vibration are given by curve 1 on Fig. 2. As seen, there is a good agreement between the two curves.

The smallest effective value u_{eff} which could be measured for $u_{\max} - u_{\min} = 12$ V and modulation depth $b = 0{,}6$ is 5.10^{-5} volts. This correspondes to an amplitude of vibration of the order of 0.01 $\overset{o}{A}$.

Using an electronic calculator it is possible to obtain the distribution of the amplitudes of vibration over the whole surface of the plate in the range of amplitudes from 0.01 to 1000 $\overset{o}{A}$.

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